From X-ray mirror surface metrology to the Point Spread Function: a self-consistent approach

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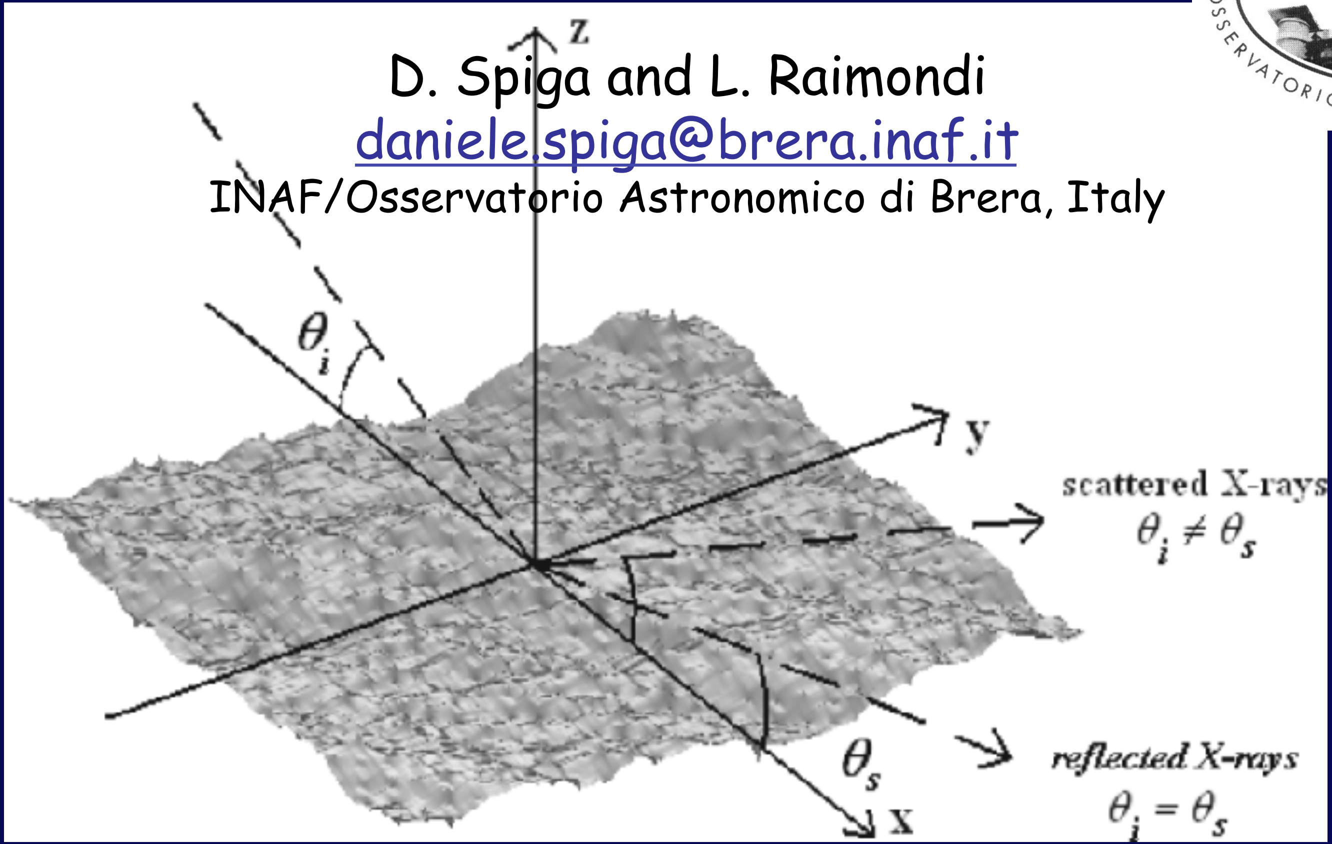
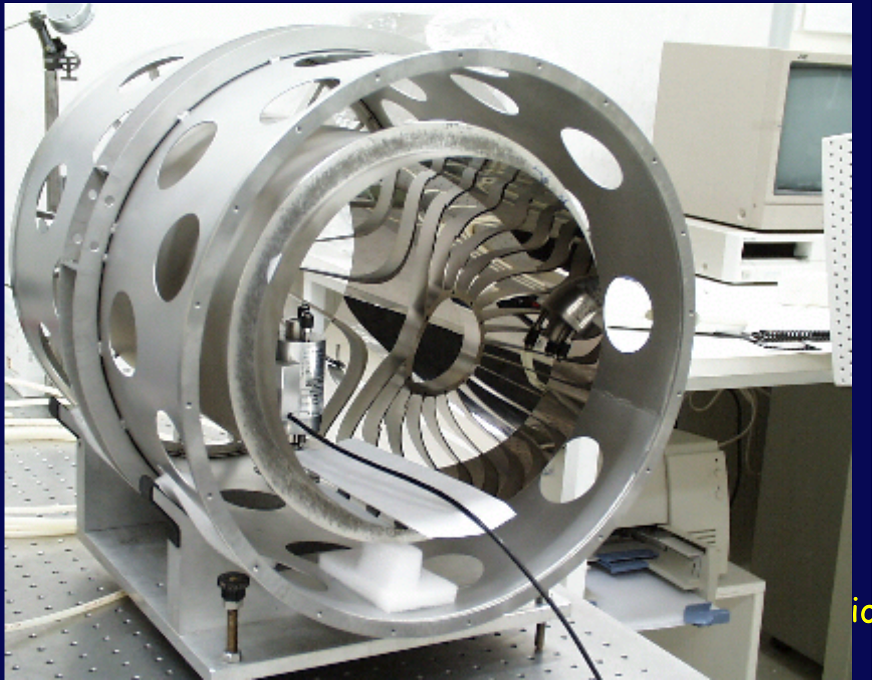
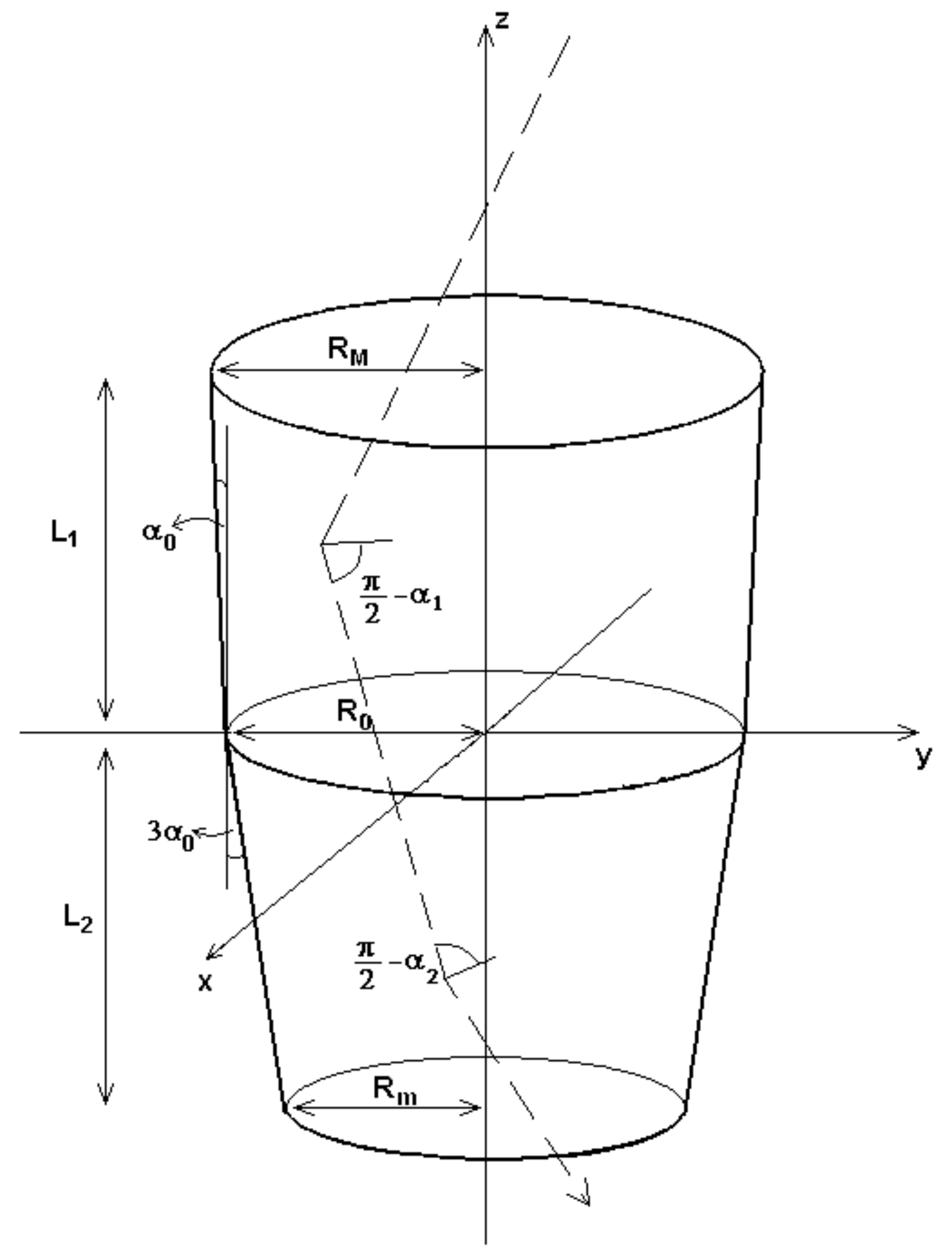


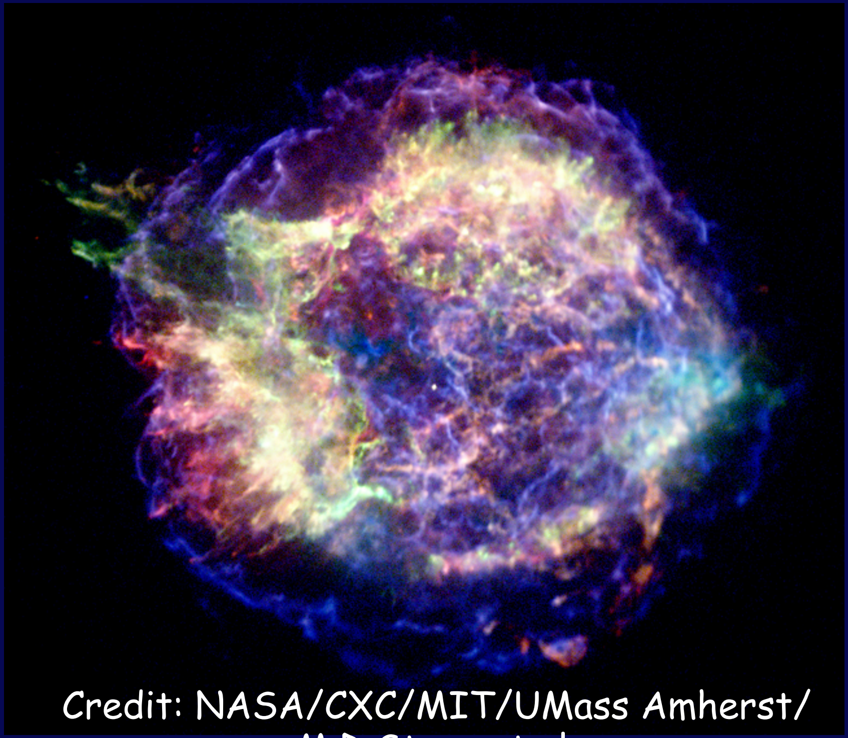
X-RAY MIRRORS FOR ASTRONOMY

- Double refl., Wolter-I (parabola +hyperbola)
- focal lengths up to tenths meter
- tight nesting
- multilayer coatings beyond 10 keV
- to be tested in full illumination
- mass production required





WHAT X-RAY ASTRONOMERS WANT: HIGH ANGULAR RESOLUTION



Credit: NASA/CXC/MIT/UMass Amherst/ M.D.Stage et al.

Credit: NASA/Swift-XRT

Chandra image, res. = 0.5 arcsec HEW

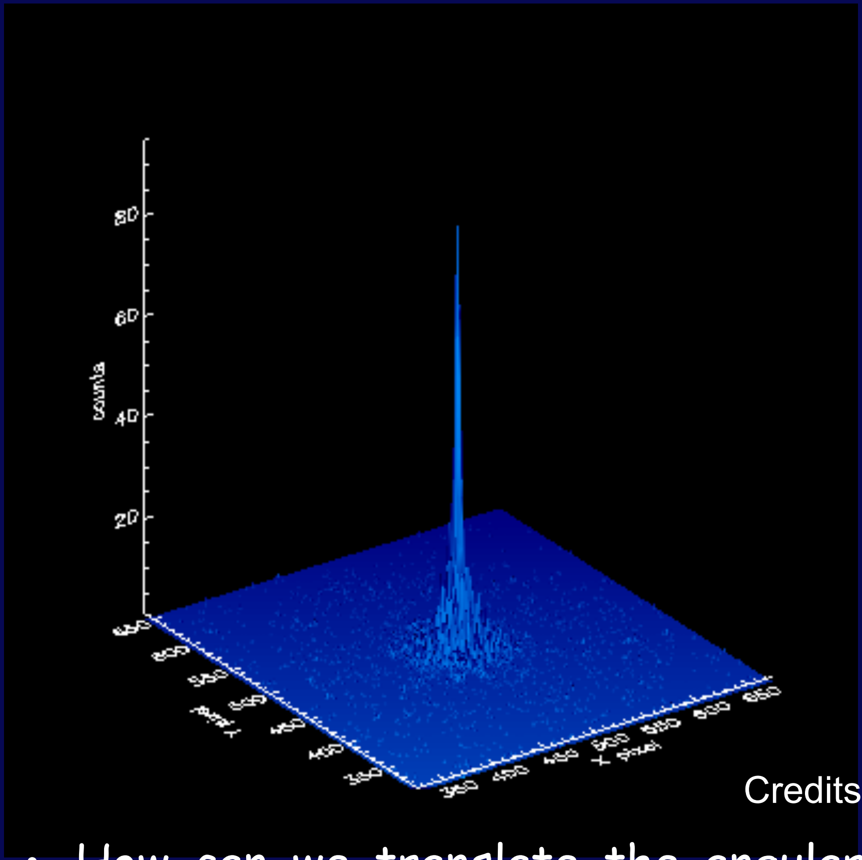
Swift XRT image, res. = 15 arcsec HEW

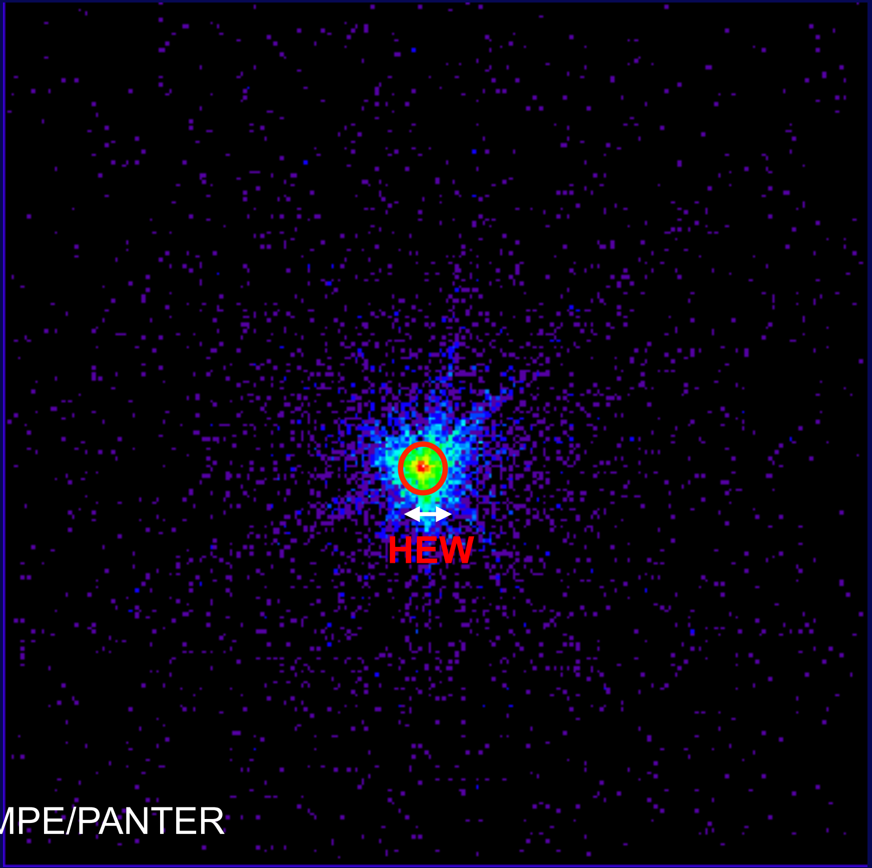
The angular resolution of X-ray telescopes is a fundamental requirement to resolve the details of celestial sources

THE ANGULAR RESOLUTION OF X-RAY TELESCOPES

The PSF (Point Spread Function) decribes how the focused intensity is spread around the focal spot.

HEW (or HPD, Half Power Diameter) = the angular diameter in arcsec including 50% of focused photons





Credits: MPE/PANTER

- How can we translate the angular resolution requirements in soft and hard X-rays into requirements to the finishing of the surface?

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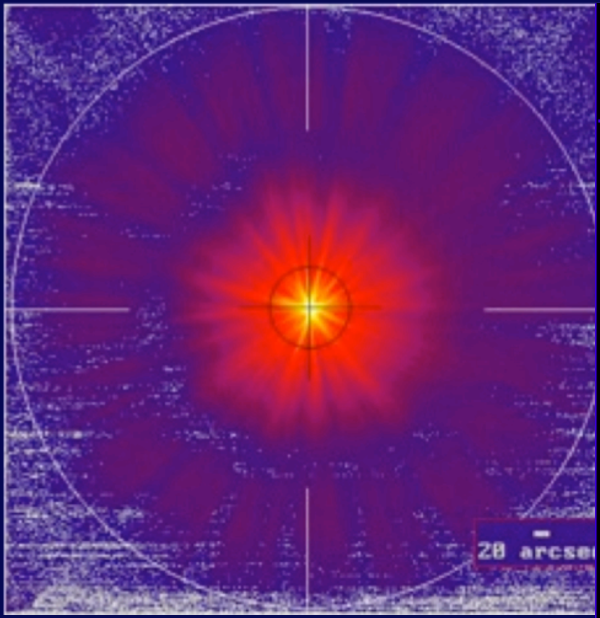
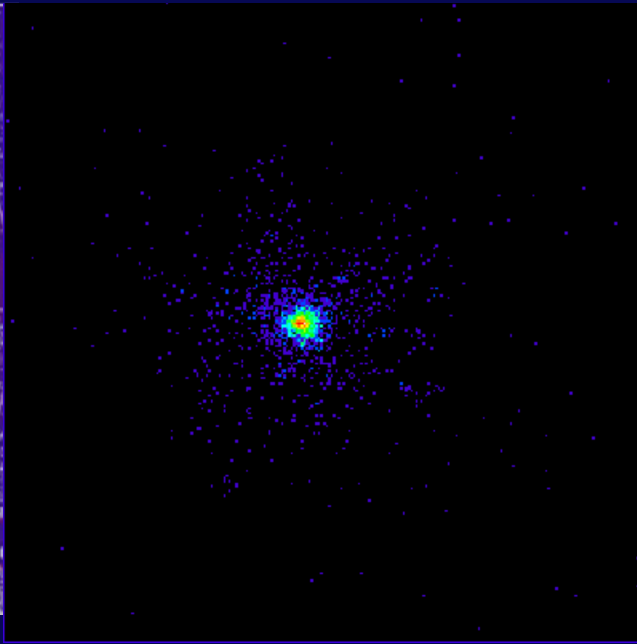
'CLASSICAL' CONTRIBUTIONS TO IMAGING DEGRADATION

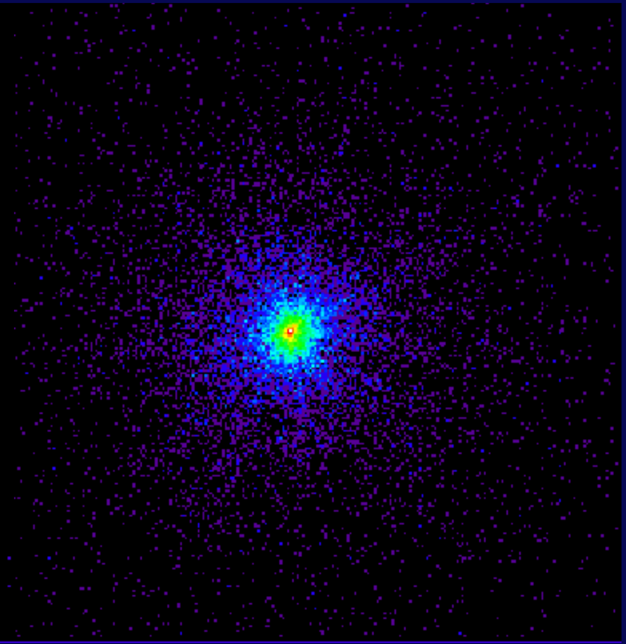
1) **Aperture diffraction** (visible in UV tests, negligible in X-rays)
2) "**Figure**" and "**slope**" errors, low and mid- spatial freq., to be treated with geometrical optics methods, figure partly seen in UV
3) **Surface roughness**, high spat freq., causing X-ray scattering (XRS, strongly energy-dependent, unseen in UV, dominant in hard X-rays)



3700 Å, 45 arcsec HEW, UV bench, INAF/OAB

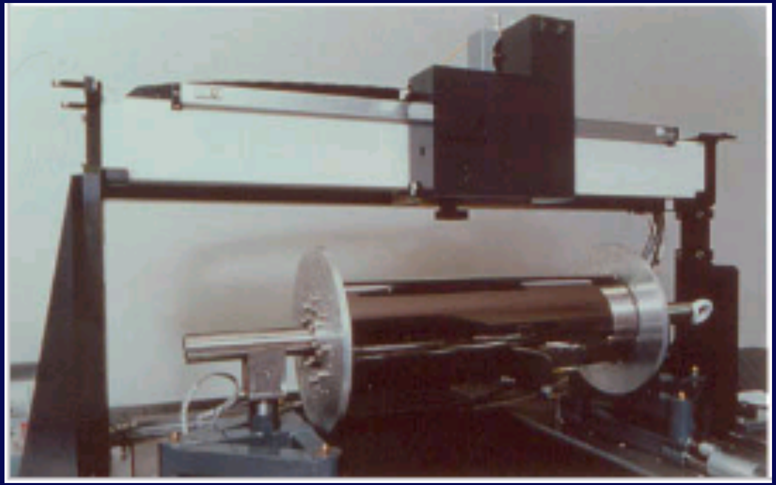


0.93 keV, 15 arcsec HEW, PANTER (MPE, Germany)

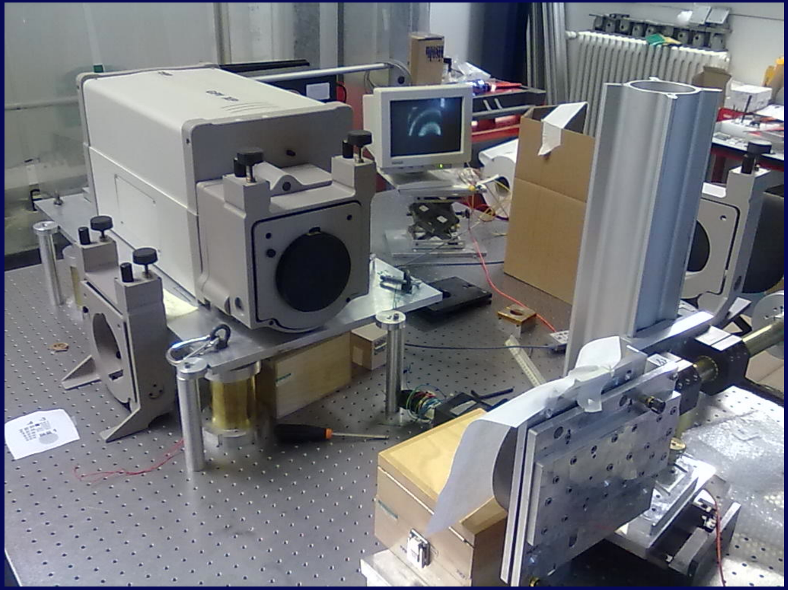


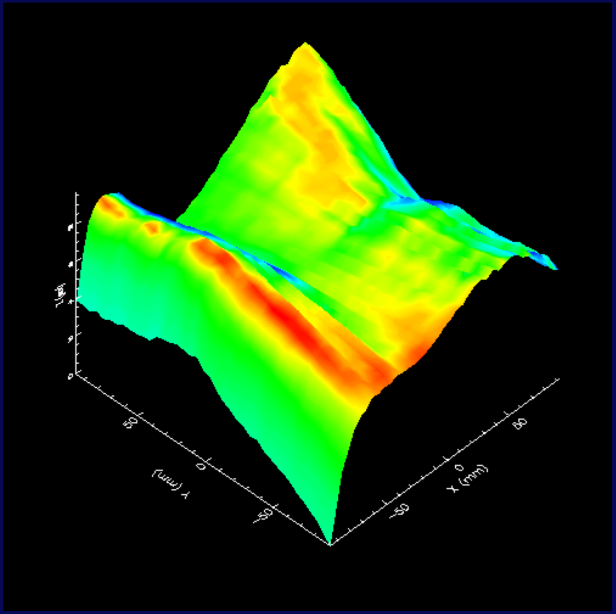
50 keV, 30 arcsec HEW, PANTER (MPE, Germany)

FIGURE ERROR MEASUREMENTS

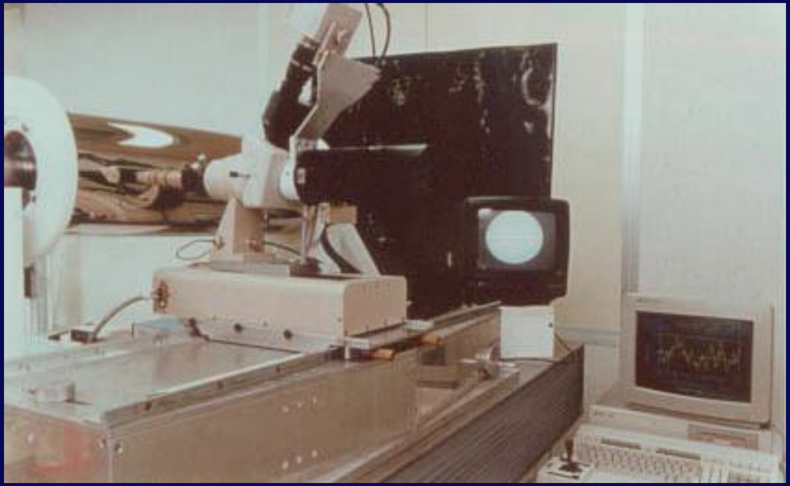


- Long-Trace profilometer for mandrels
- ZEISS contact profiometer
- ZYGO interferometer
- Home-made 3D machine (under development)
- 3D mandrel profiometer and mirror shell profilometer at the partner company Media-Lario.

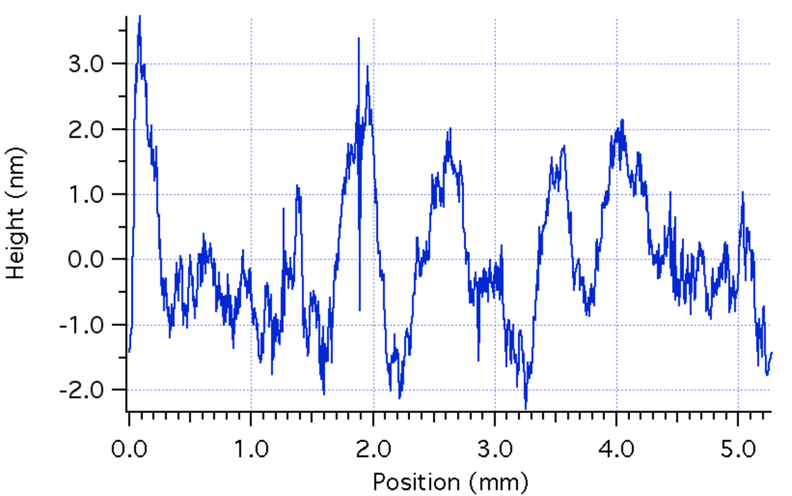




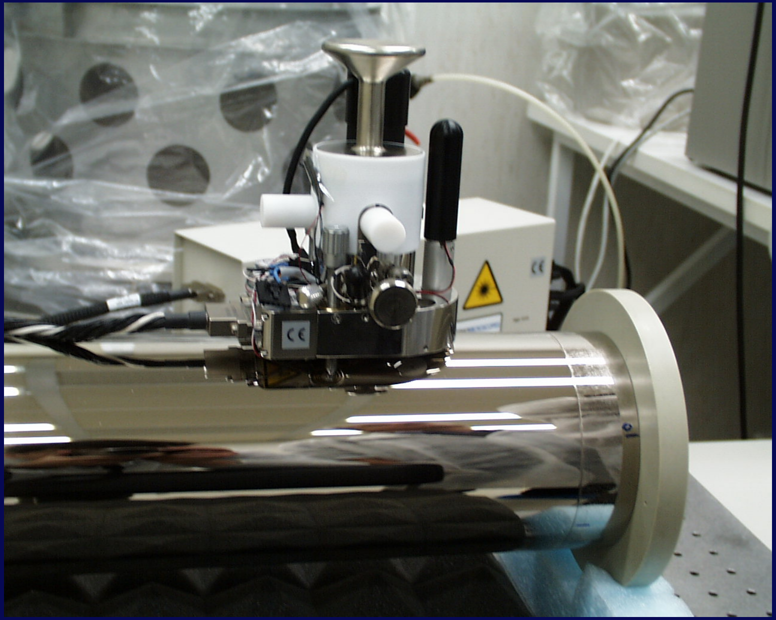
ROUGHNESS MEASUREMENTS



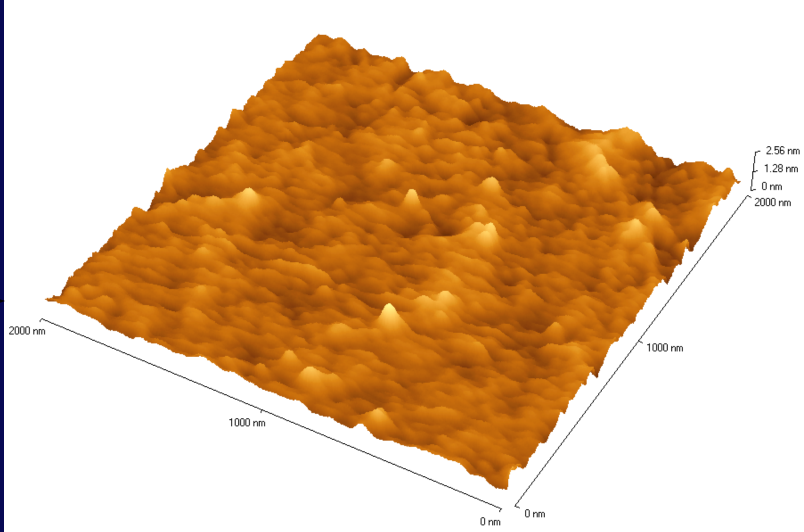
WYKO optical interferometer



5.2, 0.6 mm wide scans



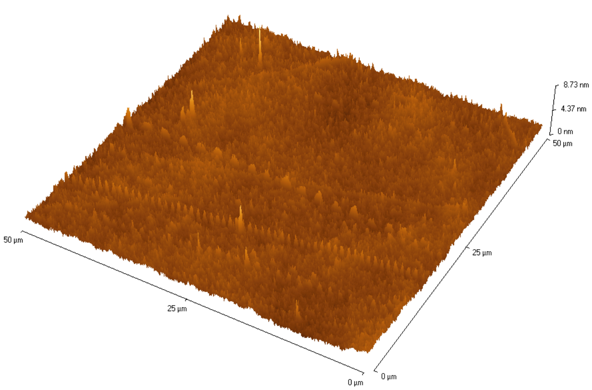
Atomic Force Microscope



100, 10, 1 µm wide scans

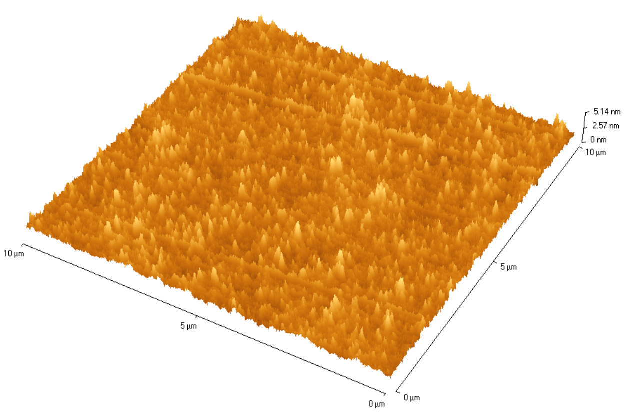
THE TREATMENT OF ROUGHNESS BY MEANS OF POWER SPECTRUM

- Each instrument is sensitive only to a particular window of spatial frequencies. The Power Spectral Density provides a global description.



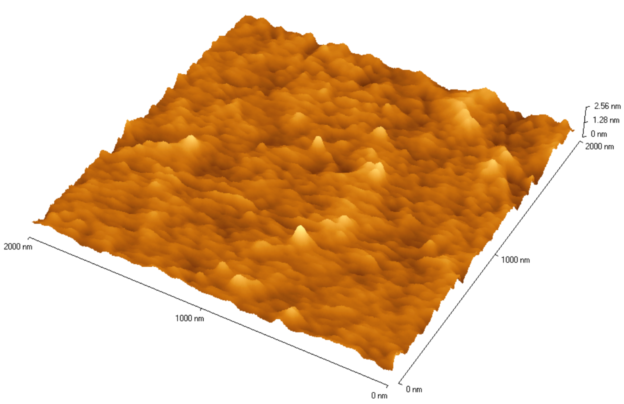
AFM, 100 µm – 0.4 µm

σ = 2.9 Å



AFM, 10 µm – 40 nm

σ = 2.6 Å



AFM, 1 µm – 4 nm

σ = 1.4 Å

☺ PSDs from different instruments are (in general) mutually-consistent

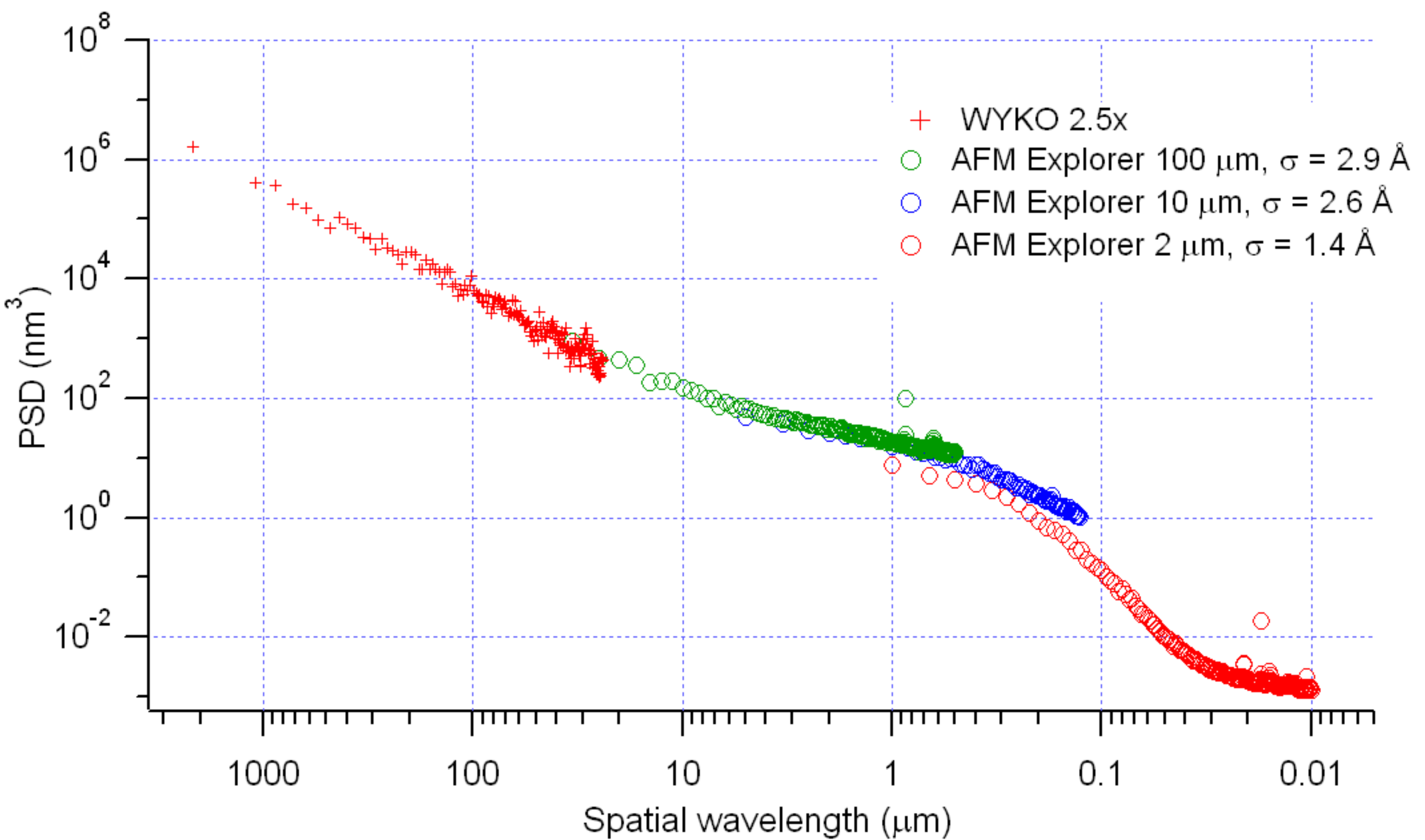
☺ PSDs in the same bandwidth can be averaged to reduce sampling effects

☺ The PSD returns a complete description of the statistical properties of roughness

☺ It directly involves the X-ray scattering and, therefore, the image degradation!

THE TREATMENT OF ROUGHNESS BY MEANS OF POWER SPECTRUM

- **Each instrument** is sensitive only to a **particular** window of spatial frequencies. The Power Spectral Density provides a global description.



OK, WE HAVE THE DATA. AND NOW?

We have now to compute the expected angular resolution in hard X-rays (hopefully, 20 arcsec or less).

1) Extrapolation of UV data? Not reliable...
    a) affected by aperture diffraction
    b) roughness not seen, mid-frequencies not seen

2) To compute
    - the figure error HEW term from profiles,
    - the HEW term due to X-ray scattering from the PSD.
    - then add them …

ANALYTICAL RELATION BETWEEN THE XRS HEW AND THE PSD

PSD ⇨ H(λ): $\int_{f_0}^{2/\lambda} P(f)\,df = \frac{\lambda^2}{16\pi^2 \sin^2\vartheta_i} \ln\left(\frac{2N}{2N-1}\right)$ → derive f_0 → $H(\lambda) = \frac{2\lambda f_0}{\sin\vartheta_i}$

H(λ) ⇨ PSD: $\frac{P(f_0)}{\lambda}\frac{d}{d\lambda}\left(\frac{H(\lambda)}{\lambda}\right) + \frac{1}{4\pi^2 \sin^3\vartheta_i} \ln\left(\frac{2N}{2N-1}\right) \approx 0$ at the freq. $f_0 = \frac{H(\lambda)}{2\lambda}\sin\vartheta_i$

N : number of identical reflections

θ_i : grazing incidence angle

λ : X-ray wavelength

f : surface spatial frequency

The method works well, however…

- the metod is based on the 1° order XRS theory.
- It requires that one can treat the figure errors and the scattering separately.

Spiga D., 2007, "*Analytical evaluation of the X-ray scattering contribution to imaging degradation ingrazing-incidence X-ray telescopes*". Astronomy and Astrophysics, vol. 468, 775–784

THE PROBLEM OF MID-FREQUENCIES

a) How should be mid-frequencies (a few mm wavelenghts) be treated? Where is the boundary between figure and roughness?

- The "Aschenbach criterion" (2005) sets a limit to the rms of single spatial frequencies that can be assumed as microroughness:

$$4\pi\sigma\sin\vartheta_i < \lambda$$

- It works with single discrete PSD spectra (see next talk) or if all the PSD integral is below this limit.

- But, what if the PSD spectrum is a continuum (e.g. most superpolished surfaces)?

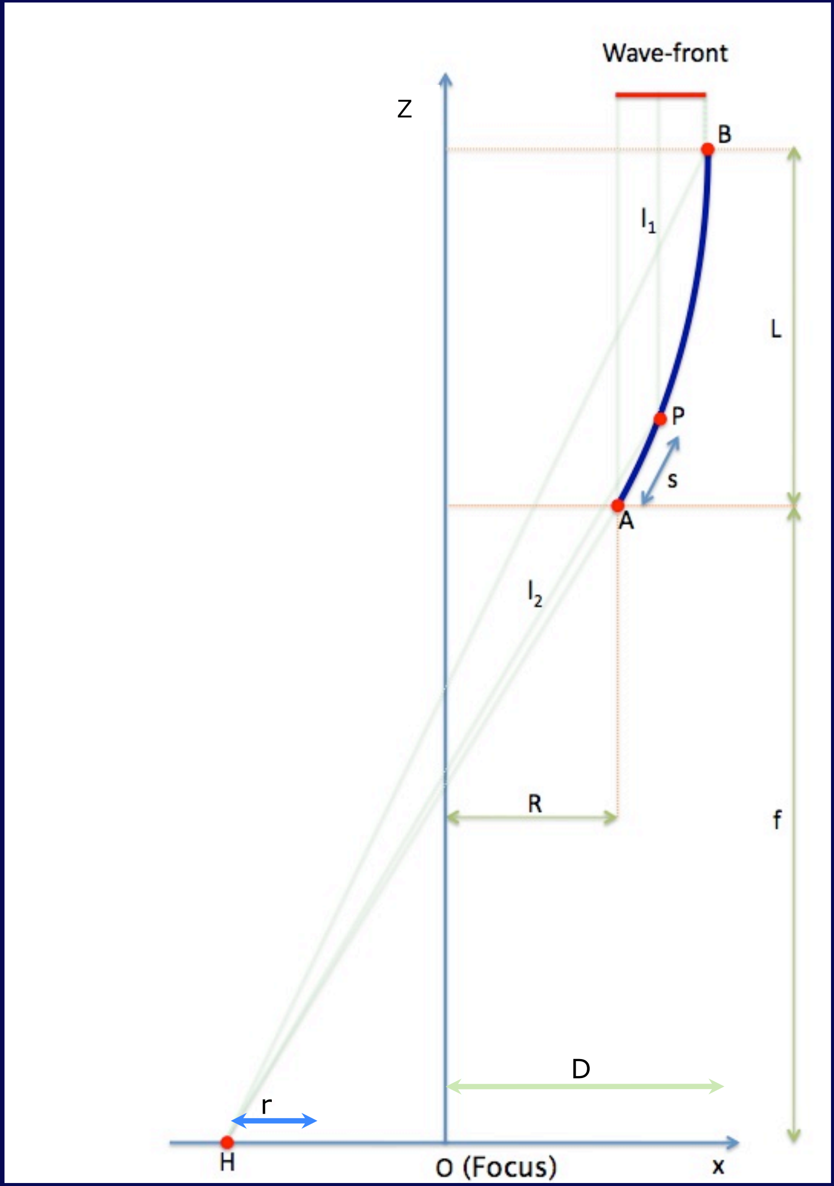
b) The boundary is not abrupt: how to treat the mid-frequencies?

c) How to mix the figure and scattering terms? …

PSF COMPUTATION FROM FRESNEL DIFFRACTION

The intensity of the beam on the focal plane is obtained from the interference of secondary waves generated at the mirror's profile of any shape (either measured or simulated).

- The method is versatile: does not need the far-field approximation (=> FFT)
- It simultaneously accounts not only for scattering, but also for figure, slope and aperture diffraction at any X-ray energy.
- It returns the PSF with the correct normalization, even if the PSF is larger than the detector !
- Setting a figure/scattering boundary is no longer needed !!



L. Raimondi, D. Spiga, *Self-consistent computation of x-ray mirror point spread functions from surface profile and roughness*, SPIE Proc., 7732 (2010)

PSF COMPUTATION FROM FRESNEL DIFFRACTION

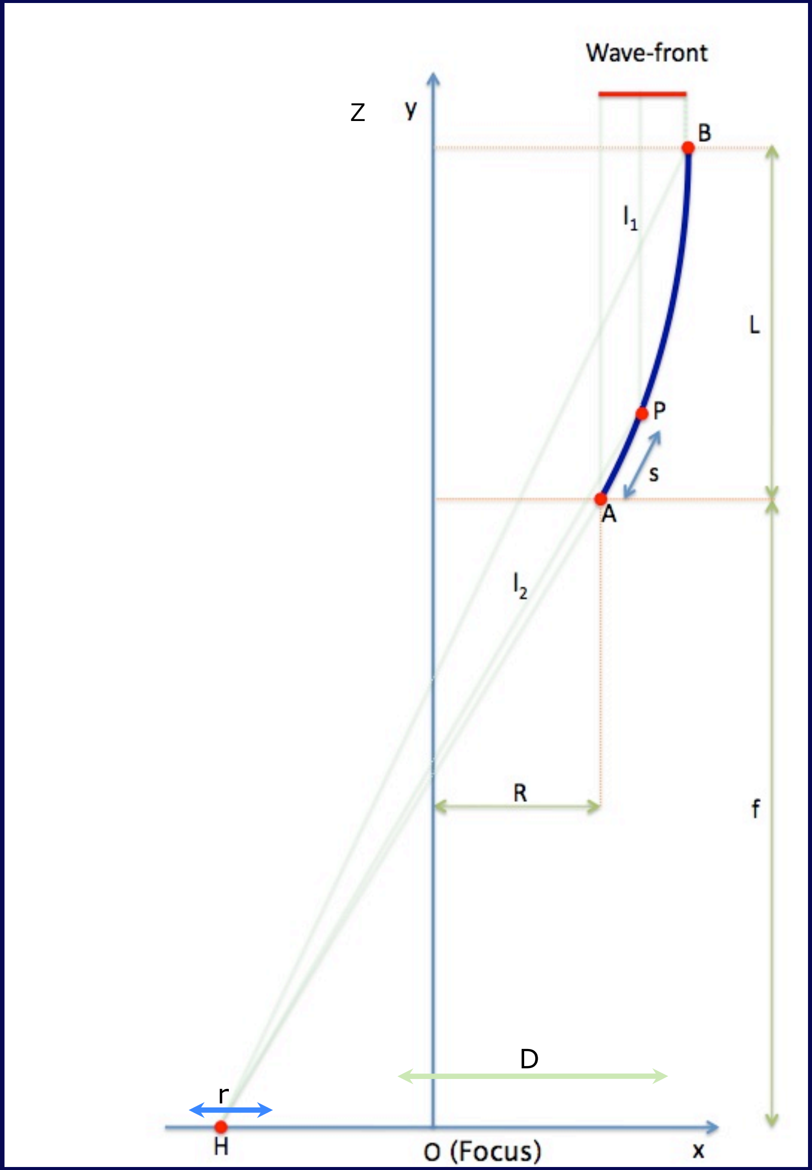
Given any profile (parabola + figure + roughness) of the mirror described by the coordinates (x_p, z_p), the PSF is computed by solving the integral:

$$PSF(x)=\frac{\Delta R}{f\lambda L^2}\left|\int_L e^{-i\frac{2\pi}{\lambda}\left(\sqrt{(x-x_p)^2+z_p^2}-z_p\right)}\,dz\right|^2$$

The minimum step of the mirror profile and the focal plane is a function of the X-ray wavelength:

$$s<\frac{\lambda f}{2\pi D\sin\alpha}$$

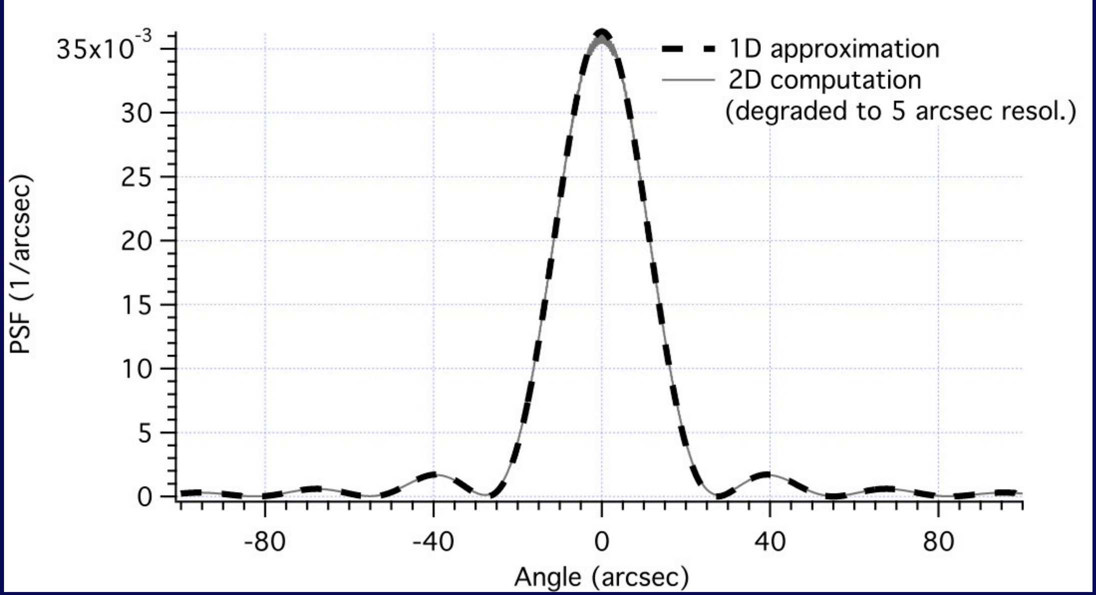
$$r<\frac{\lambda f}{2\pi L\sin\alpha}$$



ASSUMED APPROXIMATIONS

- Scalar approximation
- We can compute the PSF from 1D profiles

- Both approximations are justified by the grazing incidence, which makes the PSF to lie in the incidence plane.



- Aperture diffraction at 3000 Å simulated with Fresnel diffraction compared with 2D computation

FINAL REMARKS

- Figure and roughness tolerances have to be established in order to fulfill the angular resolution requirements of mirrors for X-ray telescopes

- The PSF prediction from metrology data is possible in a completely consistent way by solving the Fresnel integral. The computation is abridged by performing the computation using only 1D axial profiles.

- Applications to several examples of mirror deformations and roughness are shown in the next talk.

- In particular, this approach allows us to isolate the spectral range that mostly affects the PSF broadening, at given wavelenght and incidence angle. This spatial range might be corrected by an active optic system.

- Comparison with experimental data will be available soon.