Calibration of a Hall Probe Array

IMMW 20

6th June 2017 | Jan Henry Hetzel | Institut für Kernphysik 4, FZ-Jülich

Presentation for HEDI - Measurement of the **HE**SR **Di**poles: U. Bechstedt, J. Böker, C. Ehrlich, I. Engin, J. Hetzel, S. Quilitzsch, H. Soltner, P. Tripathi

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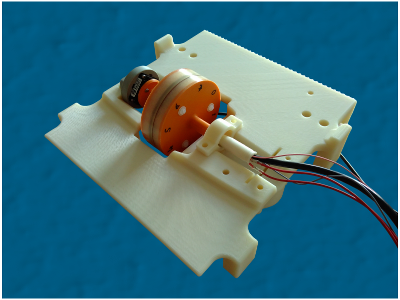
The "Rotating Hall Probes" Device



- Purpose: measurement of multipole components
- Array of 8 3D-Hall probes
- Rotatable by Piezo-Motor
- Diameter of Disc 80 mm
- Technical details: (Talk to us, we're here)
- Problem: Conventional calibration methods not possible (dimension of device!)

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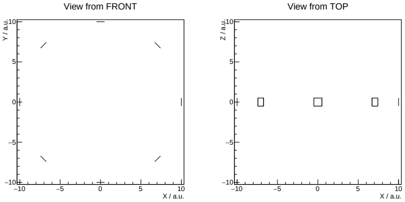

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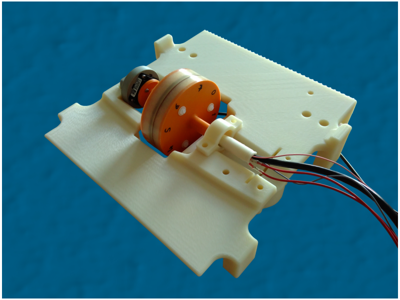

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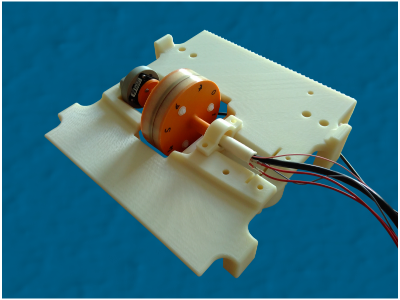

This talk: Just give an idea of used calibration method. So just probes in radial direction, up to first order.

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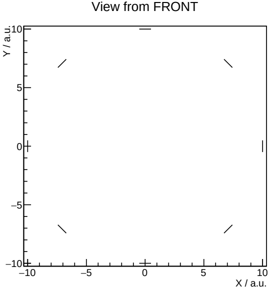
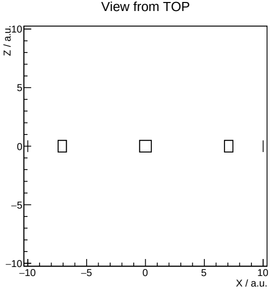
Objective of Calibration Campaign

Estimate relevant quantities





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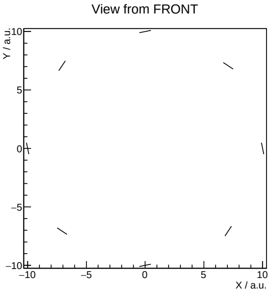

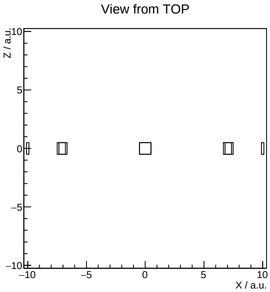

Inner Quantities

$$(i = 0, .., N - 1)$$

- Roll angle α_i

Objective of Calibration Campaign
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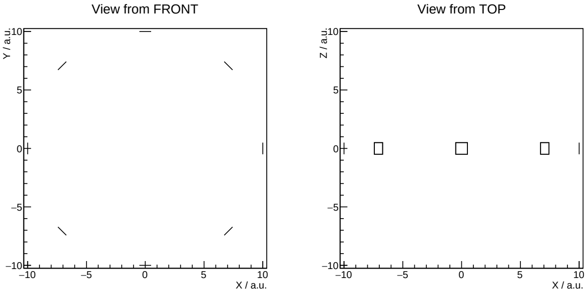

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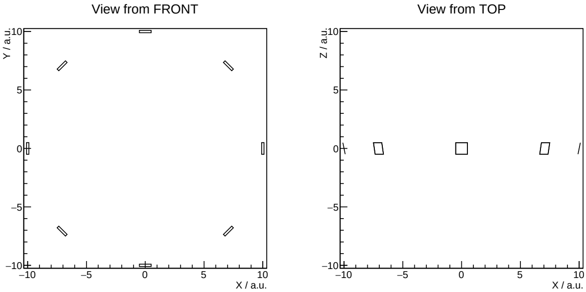

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- Pitch angle β_i

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Objective of Calibration Campaign

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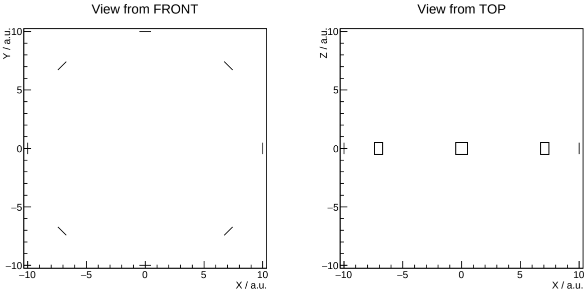

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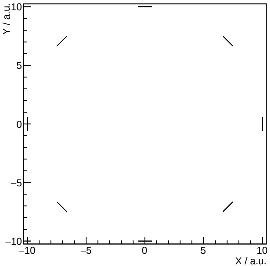
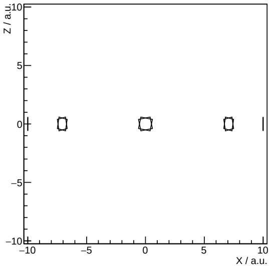
View from FRONT



View from TOP



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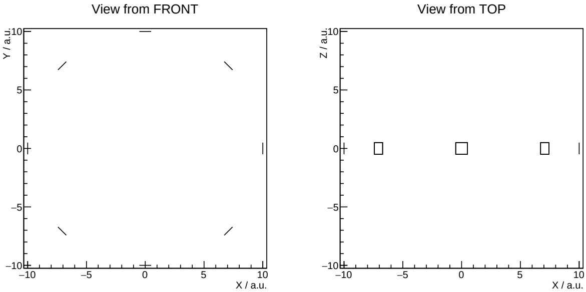
$(i = 0, .., N - 1)$

- Roll angle α_i
- Pitch angle β_i
- Yaw angle γ_i
- proportionality factor $u_i := B/U_i$

Objective of Calibration Campaign

Estimate relevant quantities



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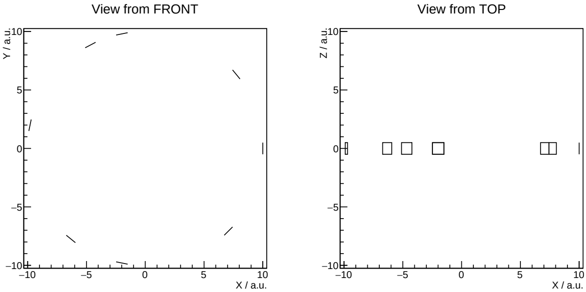

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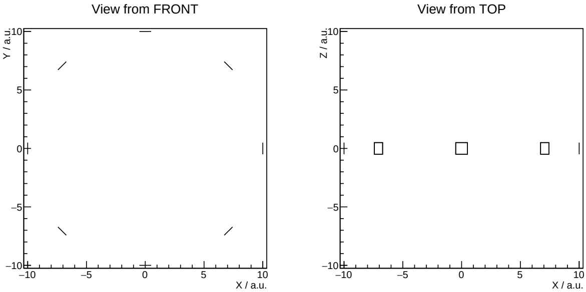

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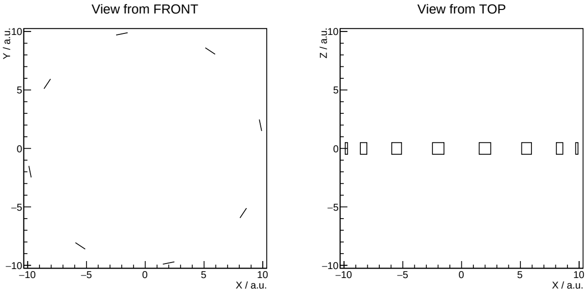

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Objective of Calibration Campaign

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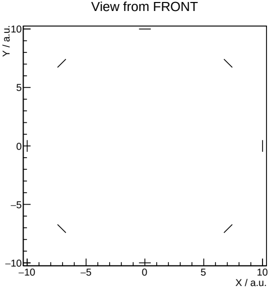

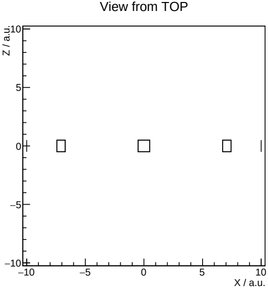

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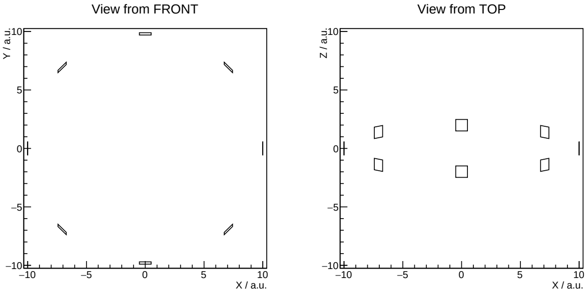

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Outer Quantities

- Roll angle A
- Pitch angle B

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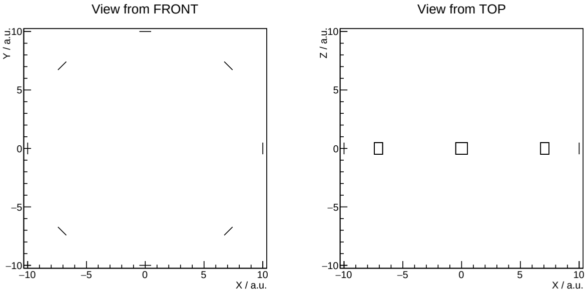

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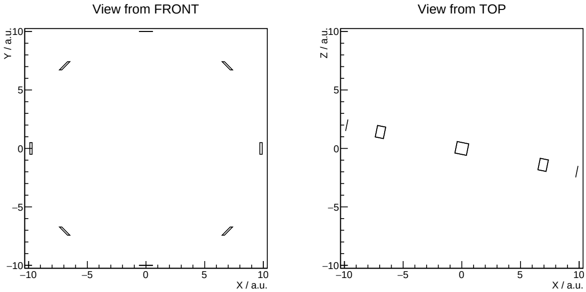

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Outer Quantities

- Roll angle A
- Pitch angle B
- Yaw angle Γ

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Objective of Calibration Campaign

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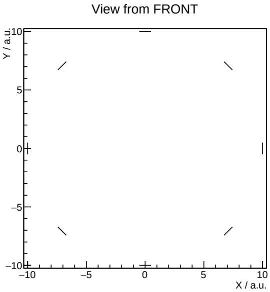

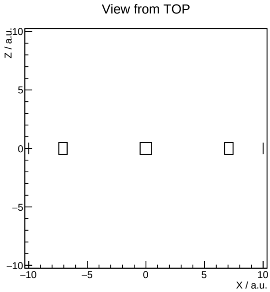

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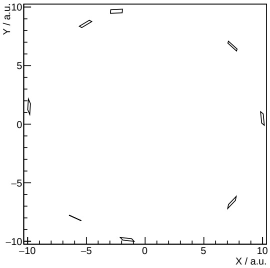

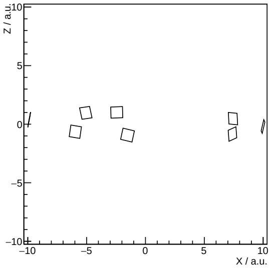

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Principle

Estimate angles from Harmonic measurements in "conventional" resistive dipoles and quadrupoles.

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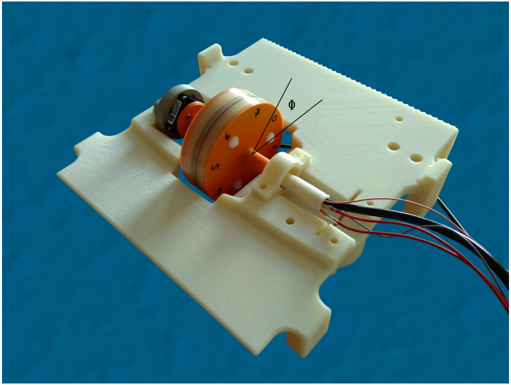
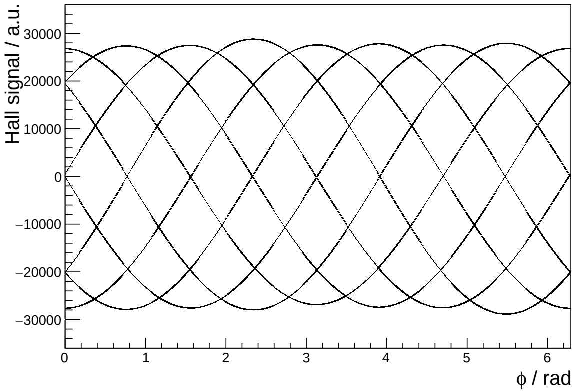


Estimate angles from Harmonic measurements in "conventional" resistive dipoles and quadrupoles.

Measurement

Example of one measurement in the dipole



all 8 probes, *r*-direction, 512 angular positions

$\Rightarrow$ needed for analysis: model of measurement

Model

Measured value $U'(\vec{x})$:

$$U'(\vec{x}) = \frac{\vec{B}(\vec{x})}{u} \cdot \vec{n}_{\mathrm{probe}}(\vec{x}) + U_0$$

proportionality factor: u
zero gauss offset: U_0
probe orientation: $\vec{n}_{\mathrm{probe}}$
B field at probe position $\vec{x}$: $\vec{B}(\vec{x})$

Model

$$U_i(\vec{x}_i) := (U'_i - U_{0,i}) = \frac{1}{u_i} B(\vec{x}_i) \cdot \vec{n}_i$$

Orientation of probe i:

$$\vec{n}_i = \mathbf{M}_{\text{yaw},\Gamma} \cdot \mathbf{M}_{\text{pitch},B} \cdot \mathbf{M}_{\text{roll},A} \cdot \mathbf{M}\left(\phi + i \cdot \frac{\pi}{4}\right) \cdot \mathbf{M}_{\varphi,i} \cdot \mathbf{M}_{\text{y.},\gamma,i} \cdot \mathbf{M}_{\text{p.},\beta,i} \cdot \mathbf{M}_{\text{r.},\alpha,i} \cdot \vec{e}$$

$$\vec{e} = (1,0,0)^T$$

Model

Order of multiplication defines interpretation of angles.

$$\vec{n}_i = \mathbf{M}_{\mathrm{yaw},\Gamma} \cdot \mathbf{M}_{\mathrm{pitch},B} \cdot \mathbf{M}_{\mathrm{roll},A} \cdot \mathbf{M}\left(\phi + i \cdot \frac{\pi}{4}\right) \cdot \mathbf{M}_{\varphi,i} \cdot \mathbf{M}_{\mathrm{y.},\gamma,i} \cdot \mathbf{M}_{\mathrm{p.},\beta,i} \cdot \mathbf{M}_{\mathrm{r.},\alpha,i} \cdot \vec{e}$$

Order is chosen such that

$$\mathbf{M}_{\mathrm{roll},A} \cdot \mathbf{M}\left(\phi + i \cdot \frac{\pi}{4}\right) \cdot \mathbf{M}_{\varphi,i}$$

can be combined to one rotation with angle $\phi + A + \varphi_i + i \cdot \frac{\pi}{4}$.

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Further simplification: Shift angle of measurements by $-i \cdot \frac{\pi}{4}$

Model

Order of multiplication defines interpretation of angles.

$$\vec{n}_i = \mathbf{M}_{\mathrm{yaw},\Gamma} \cdot \mathbf{M}_{\mathrm{pitch},B} \cdot \mathbf{M}_{\mathrm{roll},A} \cdot \mathbf{M}\left(\phi\right) \cdot \mathbf{M}_{\varphi,i} \cdot \mathbf{M}_{\mathrm{y.},\gamma,i} \cdot \mathbf{M}_{\mathrm{p.},\beta,i} \cdot \mathbf{M}_{\mathrm{r.},\alpha,i} \cdot \vec{e}$$

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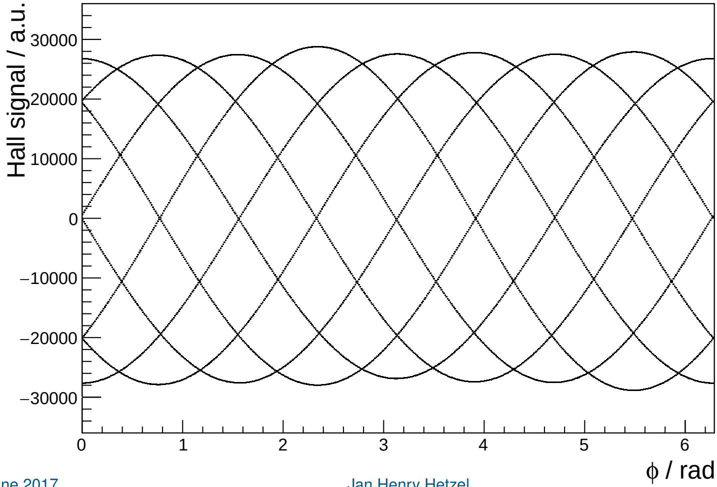
Measurement

Example of one measurement in the dipole



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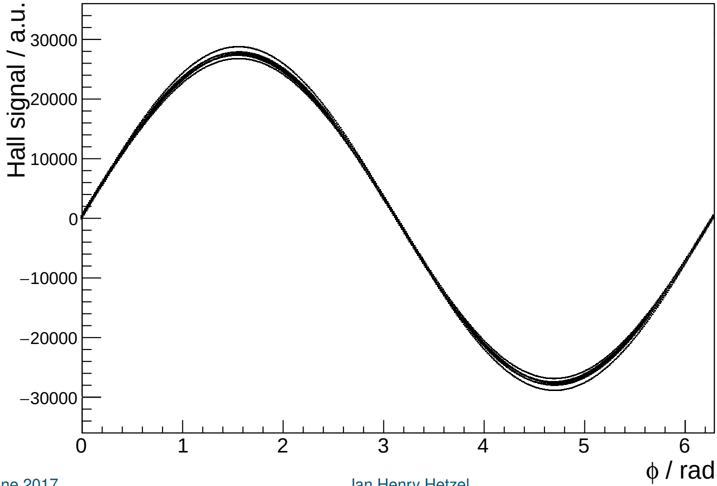

Model

$$U_i(\phi) = \frac{1}{u_i} \vec{B}(\vec{x}_i(\phi)) \cdot \vec{n}_i(\phi)$$

description of $\vec{B}$:

perfect dipole: $\vec{B}_1 = (0, B, 0)^{\mathrm{T}}$

perfect quadrupole: $\vec{B}_2 = g \cdot \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \cdot \vec{x}(\phi)$

perfect sextupole: $\vec{B}_3 = m \cdot \left(2x_1(\phi), x_1^2(\phi) + x_2^2(\phi), 0\right)^{\mathrm{T}}$

more realistic dipole: $\vec{B} = \vec{B}_1 + \vec{B}_3$

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Model

$$U_i(\phi) = \frac{1}{u_i}\vec{B}(\vec{x}_i(\phi)) \cdot \vec{n}_i(\phi)$$

Position of probe i:

$$\vec{x}_i(\phi) = \mathbf{M}_{\mathrm{yaw},\Gamma} \cdot \mathbf{M}_{\mathrm{pitch},B} \cdot \mathbf{M}\,(A + \phi + \varphi_i) \cdot \begin{pmatrix} R + r_i \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

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Model

$$U_i(\phi) = \frac{1}{u_i}\vec{B}(\vec{x}_i(\phi)) \cdot \vec{n}_i(\phi)$$

This is now to be expressed in Fourier coefficients:

$$U_i(\phi) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\phi) + b_k \sin(k\phi)$$

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Simplifying assumption: all imperfections are of same order of magnitude ξ
$\rightarrow$ use Taylor expansion up to order $\mathcal{O}(\xi^n)$

Fourier Coefficients

Radial probes, first order $\mathcal{O}(\xi)$

(perfect) Dipole:

$$a_{0,D,i} = 0$$

$$a_{1,D,i} = (A + \varphi_i + \alpha_i) \cdot \frac{B}{u_i}$$
$$b_{1,D,i} = 1 \cdot \frac{B}{u_i}$$

Quadrupole:

$$a_{0,Q,i} = 0$$

$$a_{1,Q,i} = \frac{g}{u_i}(y_0 + (A' + \varphi_i + \alpha_i) \cdot x_0)$$
$$b_{1,Q,i} = \frac{g}{u_i}(-(A' + \varphi_i + \alpha_i) \cdot y_0 + x_0)$$

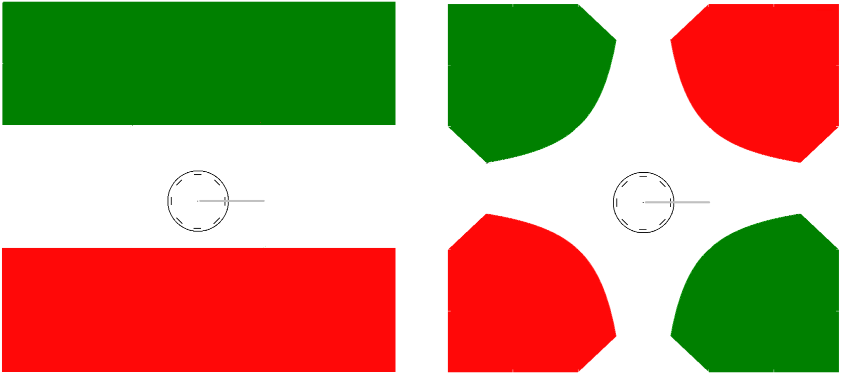
$$a_{2,Q,i} = \frac{g}{u_i}(R + r_i) \cdot (2A' + 2\varphi_i + \alpha_i)$$
$$b_{2,Q,i} = \frac{g}{u_i}(R + r_i)$$

remaining task

Combine measured Fourier coefficients to estimate angles.

Caveat

Mismatch of "outer" roll angle A



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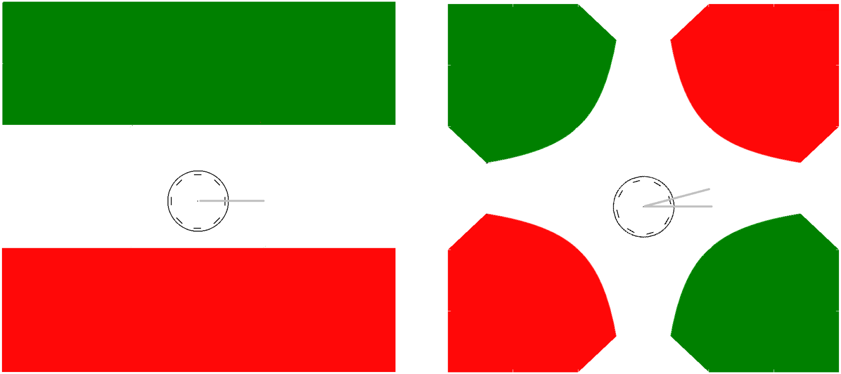

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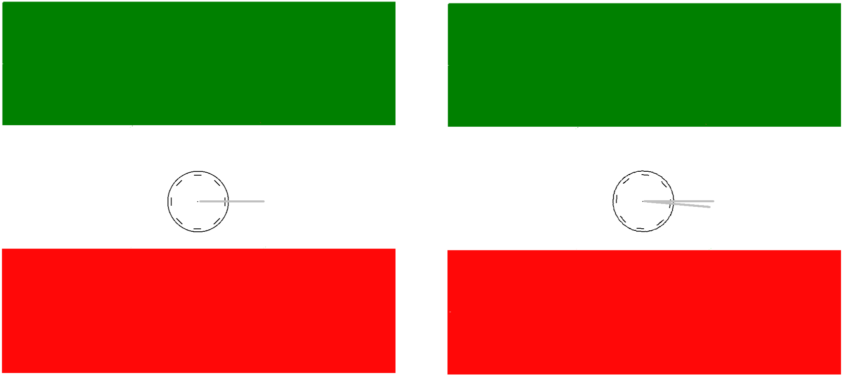


Solution

Use field to find orientation

Fourier coefficients

Dipole:
$a_{1,D,i} = (A + \varphi_i + \alpha_i) \cdot \frac{B}{u_i}$
Quadrupole:
$a_{1,Q,i} = \frac{g}{u_i}(y_0 + (A' + \varphi_i + \alpha_i) \cdot x_0)$

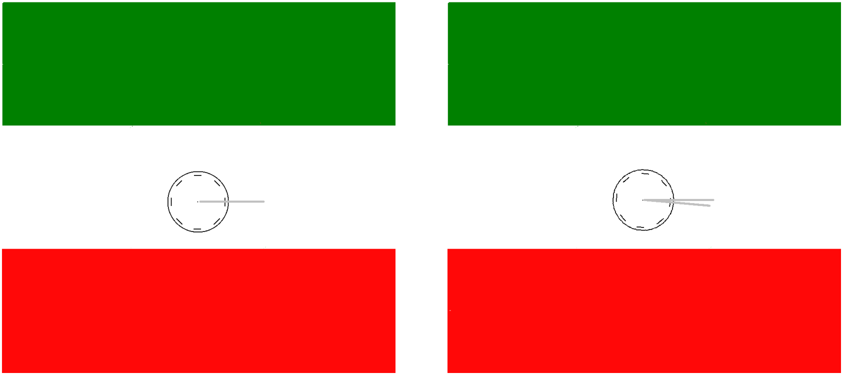
Reminder: Rotation around longitudinal axis $\propto A + \varphi_i + \phi$
$\Rightarrow$ Mismatch of A can be "absorbed" in initial ϕ

Shift angle of measurements, such that in Dipole:

$$a_{1,D,0} = 0$$

Problem solved I

Mismatch of "outer" roll angle A in dipole



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Quadrupole:

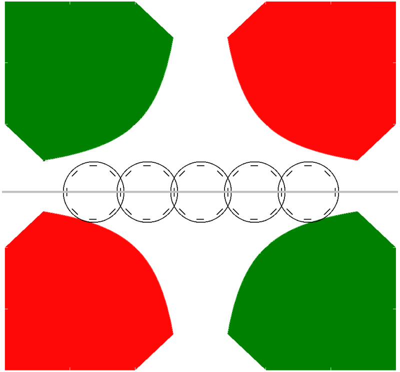
$$a_{1,Q,i} = \frac{g}{u_i}(y_0 + (A' + \varphi_i + \alpha_i) \cdot x_0)$$

Wanted: $A = A'$

already aligned: $a_{1,D,0} = 0$ Consequently: $a_{1,Q,0} \overset{!}{=} \frac{g}{u_i} y_0$ or

$$a_{1,Q,0}(x_0) = \text{const.}$$

Multiple Measurements at Different Horizontal Positions



choose initial ϕ such, that
$a_{1,Q,0} = \text{const.}$
Boundary condition:

$$\frac{a_{2,Q,0}}{b_{2,Q,0}} = 2A' + \alpha_0$$

same for all measurements.

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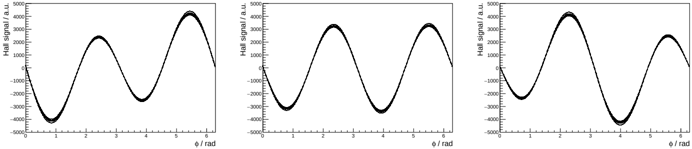
Multiple Measurements at Different Horizontal Positions

Example

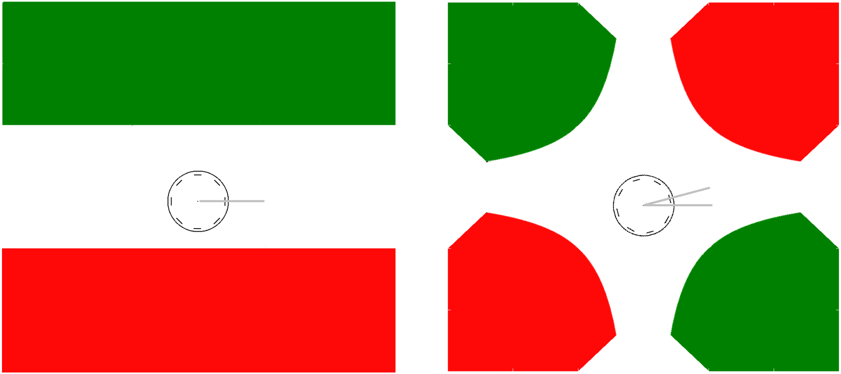


Desired initial ϕ can be found by

$$\phi = -\frac{\frac{\partial a_{1,Q,0}}{\partial x_0}(x_0)}{\frac{\partial b_{1,Q,0}}{\partial x_0}(x_0)} = -(A' + \alpha_0)$$

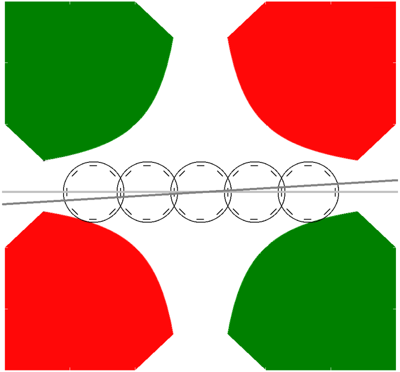
Problem Solved II?

Mismatch of "outer" roll angle A in dipole and quadrupole



Problem Solved II?

Main source of systematic errors



- displacement has to be carefully matched to horizontal axis
- rely on just a few measurements
- higher order multipoles especially in the outer regions present

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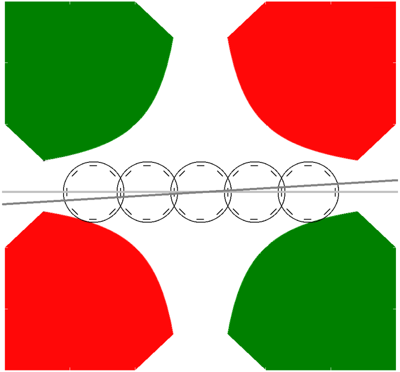

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I Want You!

Better ideas of how to align measurements in dipole and quadrupole welcome.

Combination of Measured Coefficients

Radial probes $\mathcal{O}(\xi)$

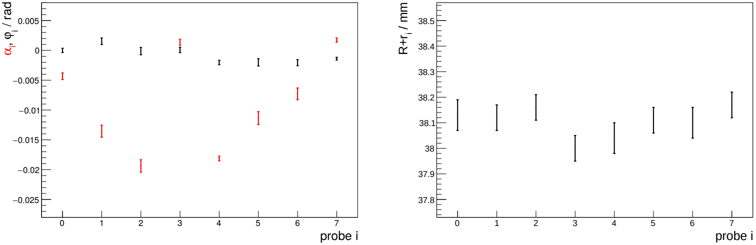
$$\alpha_i = 2\frac{a_{1,D,i}}{b_{1,Di}} - \frac{a_{2,Q,i}}{b_{2,Q,i}}$$

$$\phi_i = \frac{a_{2,Q,i}}{b_{2,Q,i}} - \frac{a_{1,D,i}}{b_{1,D,i}} + \alpha_0$$

$$R + r_i = \frac{b_{2,Q,i}}{\frac{\partial b_{1,Q,i}}{\partial x_0}(x_0)}$$

Knowledge of proportionality factors u_i not needed at this stage.
Redundancy between A and one out of 3×8 $\varphi_i \Rightarrow$ set $\varphi_0 = 0$ for radial probe.

Results
Radial probes



Systematic Uncertainties

- During measurement: "Flip and repeat"
- $\xi_{max} \approx 0.02$ rad $\Rightarrow \mathcal{O}(\xi^2) \approx 5 \cdot 10^{-4}$ rad
- sextupole contribution, $\mathcal{O}(\xi)$):

$$a_1 \cdot u = m \cdot \left(2xy + (A + \varphi + \alpha)(x^2 + y^2) + (R + r)^2 A\right)$$

$$b_3 \cdot u = m \cdot (R + r)^2$$

even for maximum measured b_3 correction to $a_1 = \mathcal{O}(x^2)$

Conclusion & Outlook

- successfully determined positions and roll angles of angular probes to first order
- determine proportionality factors u_i by comparison with NMR measurements
- determine remaining angles by using second order
- determine remaining angles by additional measurements in solenoid

Acknowledgement

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