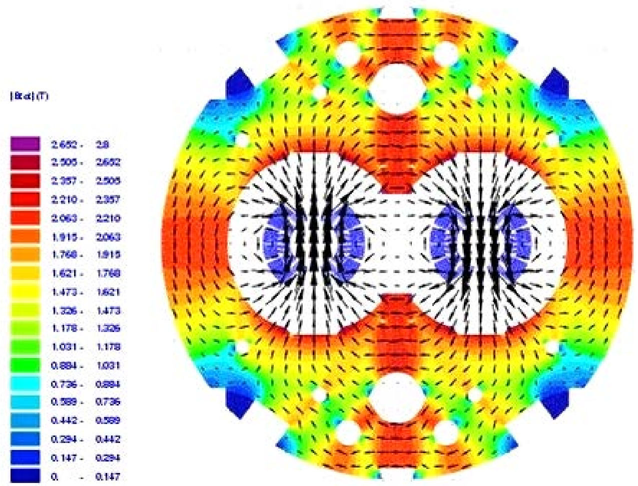
Rotating-Coil Scanner (Toy Train) and the Translating Fluxmeter for Extracting Pseudo Multipoles in Accelerator Magnets

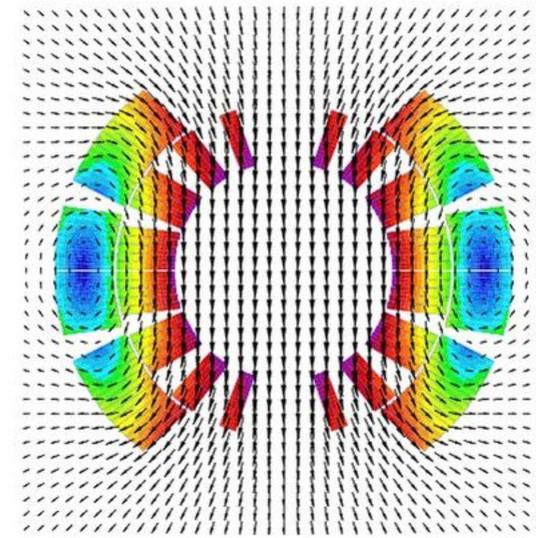
S. Russenschuck
CERN TE-MSC-MM

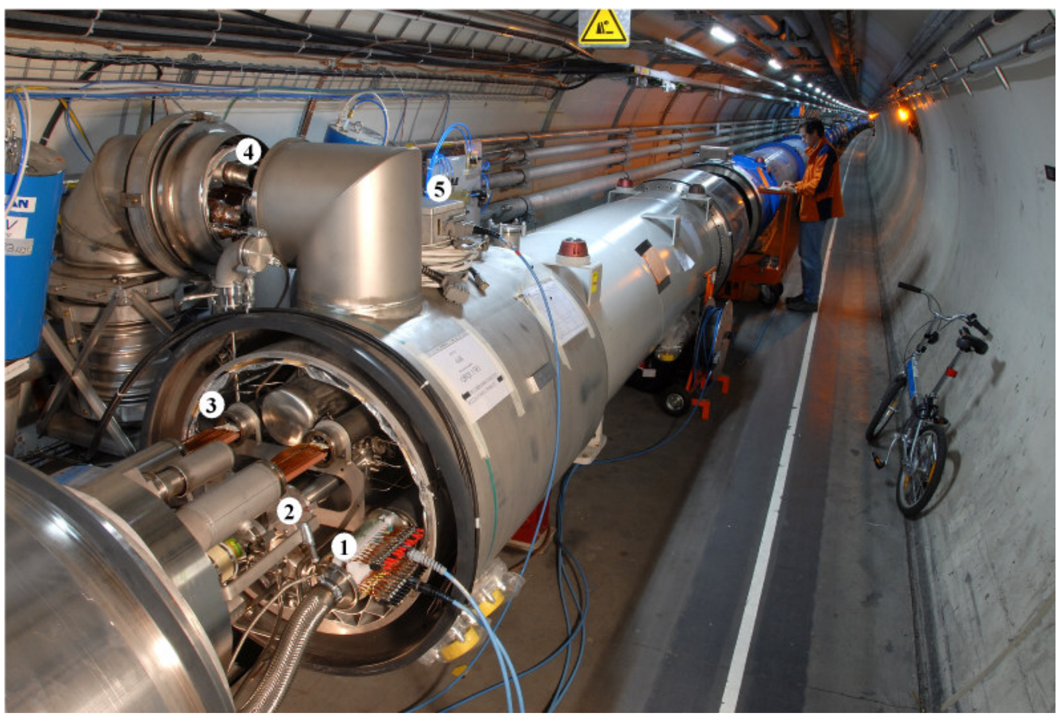
7.06.2017

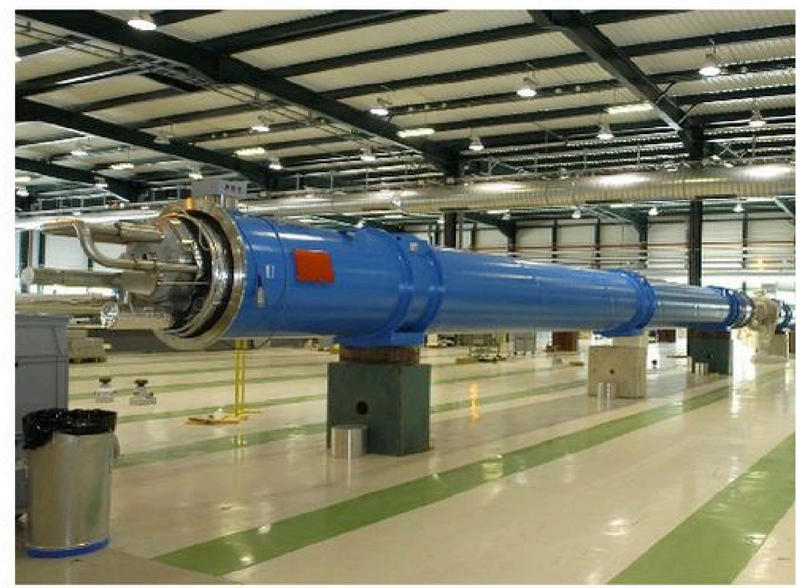
Superconducting LHC Magnets

Round aperture
Long and nearly straight (2D)
Coil dominated



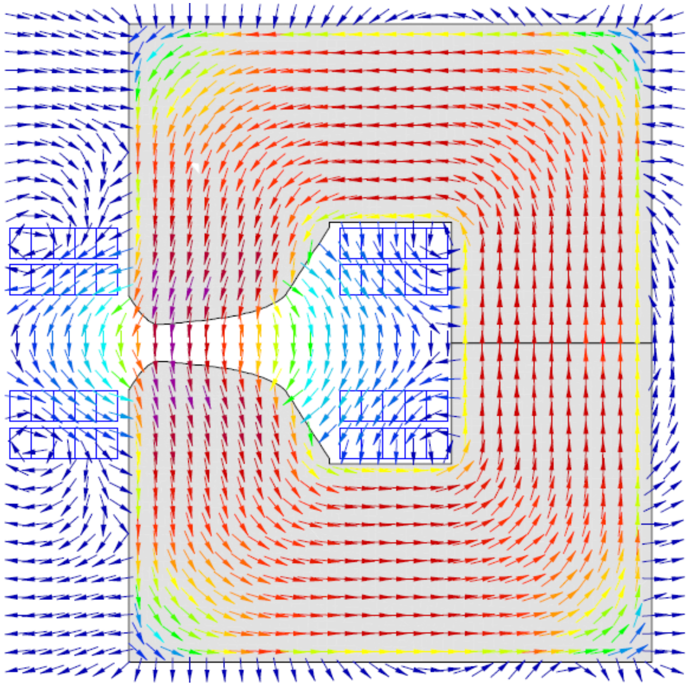






Iron Dominated Magnets

Combined function
Curved
Fast ramped

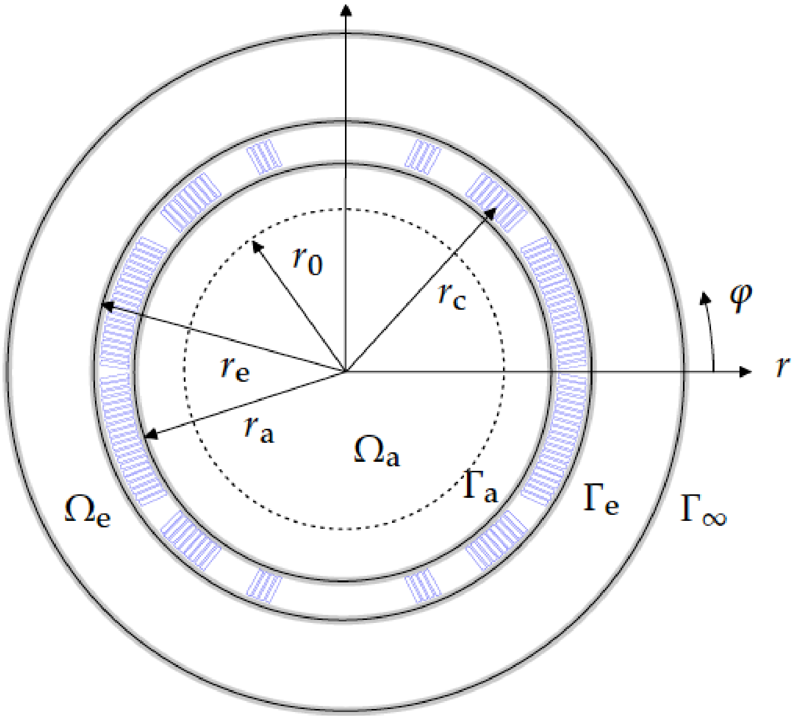


3D Eddy currents and hysteresis
-> strange wipe-out properties

1. Governing equation in the air domain

$$\nabla^2 A_z = 0,$$

2. Chose a suitable coordinate system



$$r^2 \frac{\partial^2 A_z}{\partial r^2} + r \frac{\partial A_z}{\partial r} + \frac{\partial^2 A_z}{\partial \varphi^2} = 0,$$

3. Make a guess, look it up in a book, use the method of separation:
That is: find eigenfunctions. Coefficients are not know yet

$$A_z(r, \varphi) = \sum_{n=1}^{\infty} r^n (\mathcal{A}_n \sin n\varphi + \mathcal{B}_n \cos n\varphi).$$

4. Derive the radial field component

$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1} (\mathcal{A}_n \cos n\varphi - \mathcal{B}_n \sin n\varphi),$$

5. Measure (or calculate) the field on a reference radius and perform Fourier analysis (develop into the eigenfunctions). Coefficients are known here.

$$B_r(r_0,\varphi) = \sum_{n=1}^{\infty} (B_n(r_0) \sin n\varphi + A_n(r_0) \cos n\varphi),$$

Solving the Boundary Value Problem

6: Compare the known and unknown coefficients

$$B_r(r,\varphi) = \frac{1}{r}\frac{\partial A_z}{\partial \varphi} = \sum_{n=1}^{\infty} n r^{n-1} (\mathcal{A}_n \cos n\varphi - \mathcal{B}_n \sin n\varphi),$$

$$B_r(r_0,\varphi) = \sum_{n=1}^{\infty} (B_n(r_0) \sin n\varphi + A_n(r_0) \cos n\varphi),$$

$$\mathcal{A}_n = \frac{1}{n\, r_0^{n-1}} A_n(r_0)\,, \qquad \mathcal{B}_n = \frac{-1}{n\, r_0^{n-1}} B_n(r_0)\,.$$

7. Put this into the original solution for the entire air domain

$$A_z(r,\varphi) = -\sum_{n=1}^{\infty} \frac{r_0}{n} \left(\frac{r}{r_0}\right)^n (B_n(r_0) \cos n\varphi - A_n(r_0) \sin n\varphi).$$

8: Calculate fields and potentials in the entire air domain

$$A_z(r,\varphi) = -\sum_{n=1}^{\infty} \frac{r_0}{n} \left(\frac{r}{r_0}\right)^n (B_n(r_0)\cos n\varphi - A_n(r_0)\sin n\varphi).$$

$$B_r(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n(r_0)\sin n\varphi + A_n(r_0)\cos n\varphi)$$

$$B_\varphi(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n(r_0)\cos n\varphi - A_n(r_0)\sin n\varphi)$$

$$B_x(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n(r_0)\sin(n-1)\varphi + A_n(r_0)\cos(n-1)\varphi)$$

$$B_y(r,\varphi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0}\right)^{n-1} (B_n(r_0)\cos(n-1)\varphi - A_n(r_0)\sin(n-1)\varphi)$$

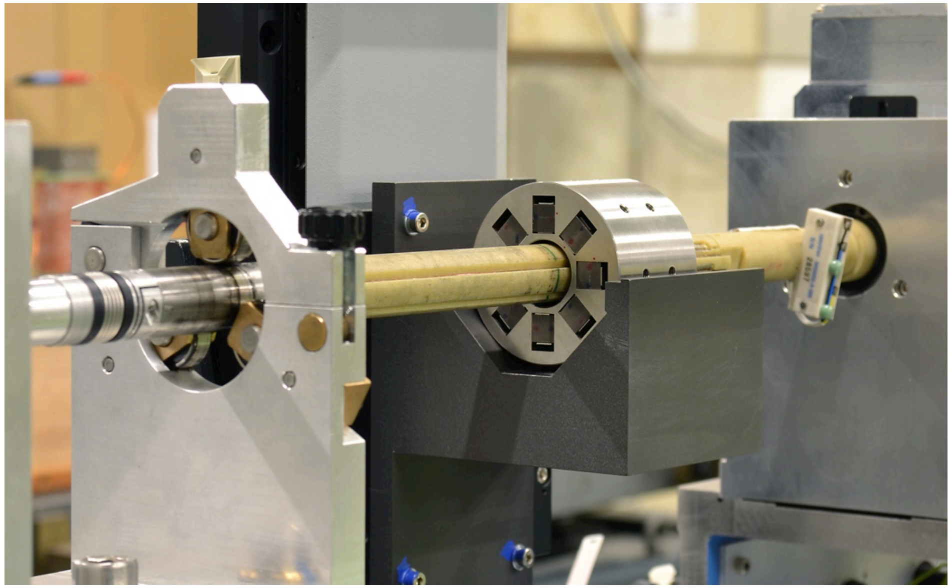
Take any 2π periodic function and develop according to

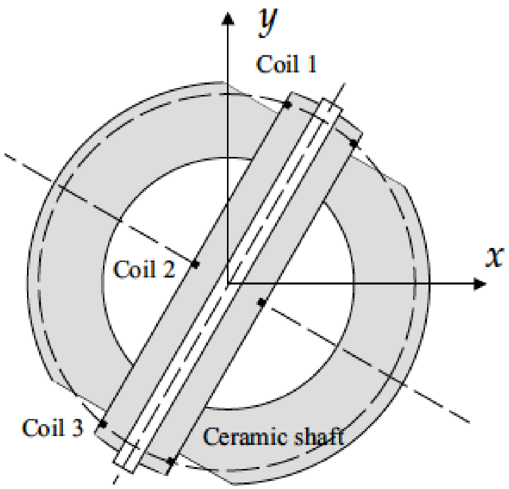
$$\frac{C_0}{2} + \sum_{n=1}^{\infty} \left(C_n(r_0) \sin n\varphi + D_n(r_0) \cos n\varphi \right).$$

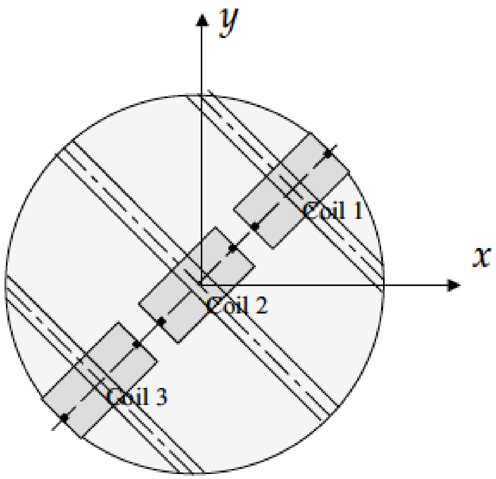
	B_r	B_φ	B_x	B_y	A_z	ϕ_{m}
$B_n =$	C_n	D_n	C_{n-1}	D_{n-1}	$\frac{-nD_n}{r_0}$	$\frac{-n\mu_0 C_n}{r_0}$
$A_n =$	D_n	$-C_n$	D_{n-1}	$-C_{n-1}$	$\frac{nC_n}{r_0}$	$\frac{-n\mu_0 D_n}{r_0}$

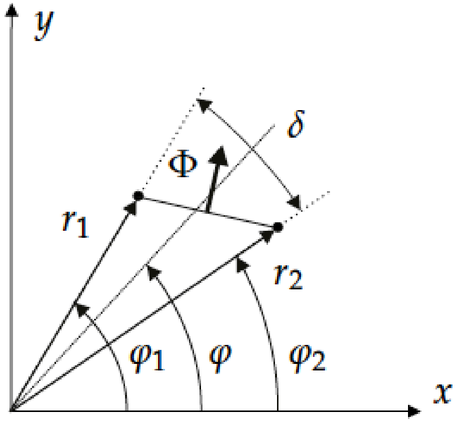
We can use fields, potentials, fluxes, or wire-oscillation amplitudes as “raw data”.

Rotating Coil Measurements



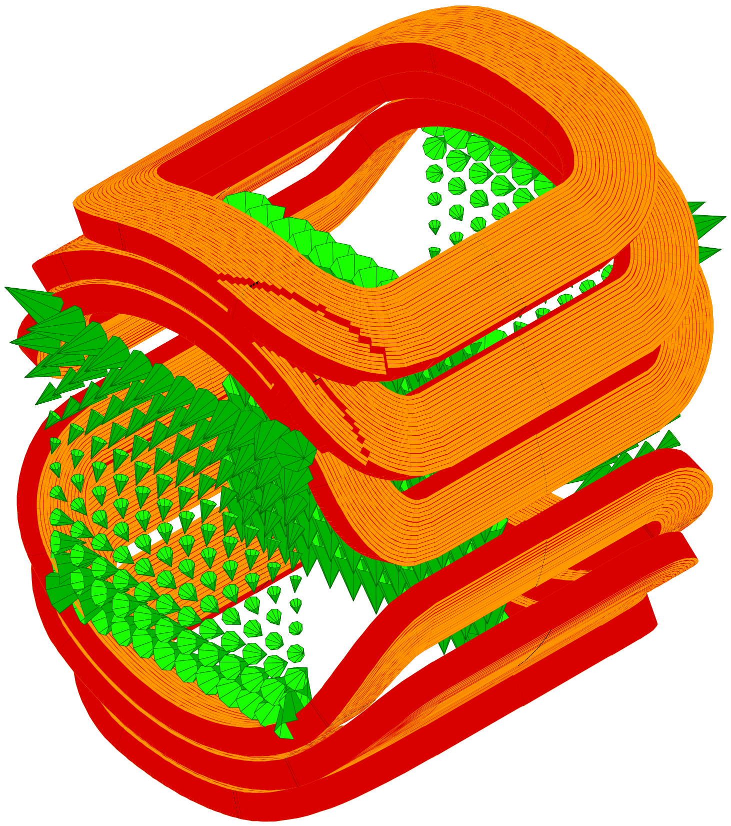


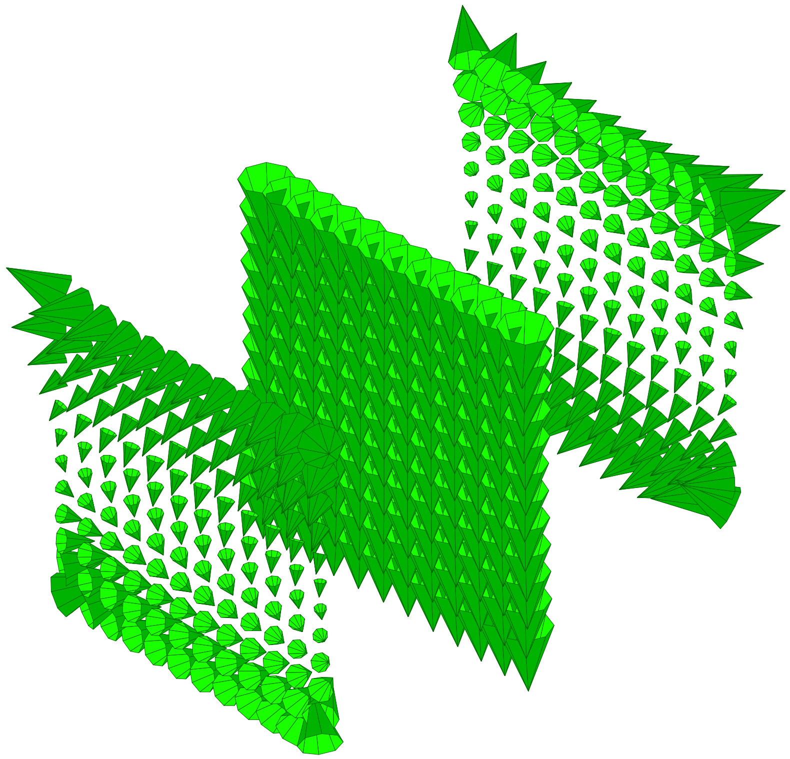


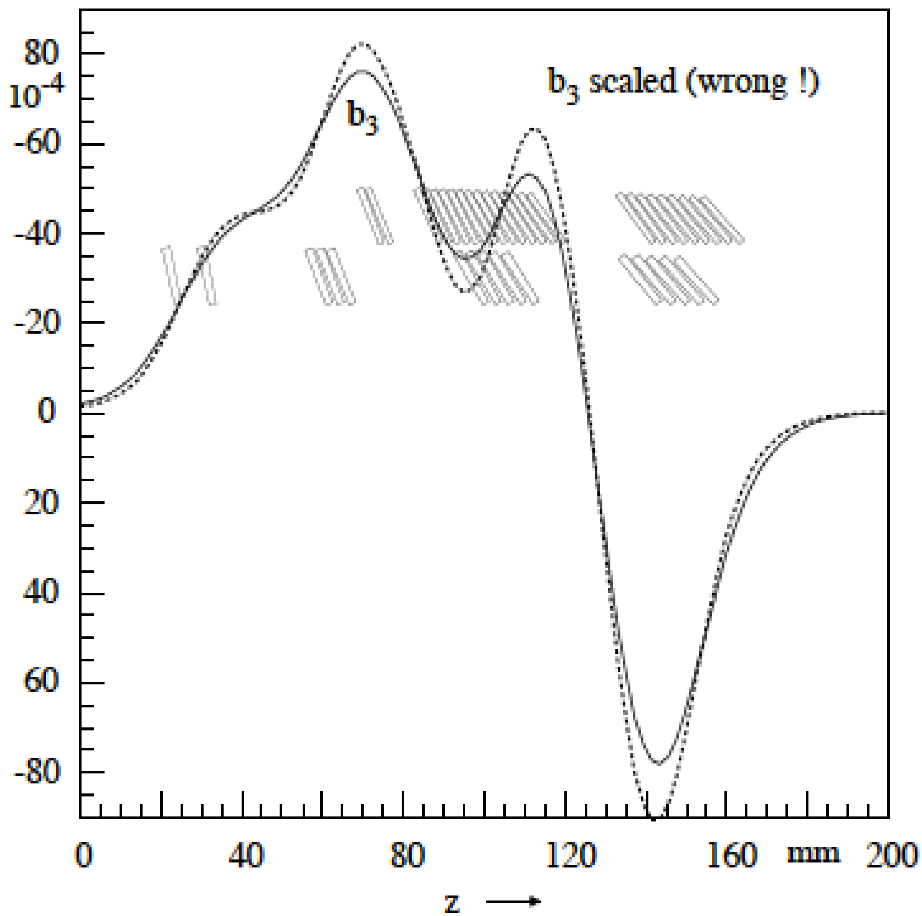


Tangential coil
Radial flux

Radial coil
Tangential flux







Local transverse harmonics calculated at different reference radii and scaled with the 2D laws

$$b_n(r_1) = \left(\frac{r_1}{r_0}\right)^{n-N} b_n(r_0),$$

wrong

Integrated Harmonics

$$\nabla^2 \phi_\mathrm{m}(x,y,z) = \frac{\partial^2 \phi_\mathrm{m}(x,y,z)}{\partial x^2} + \frac{\partial^2 \phi_\mathrm{m}(x,y,z)}{\partial y^2} + \frac{\partial^2 \phi_\mathrm{m}(x,y,z)}{\partial z^2} = 0\,.$$

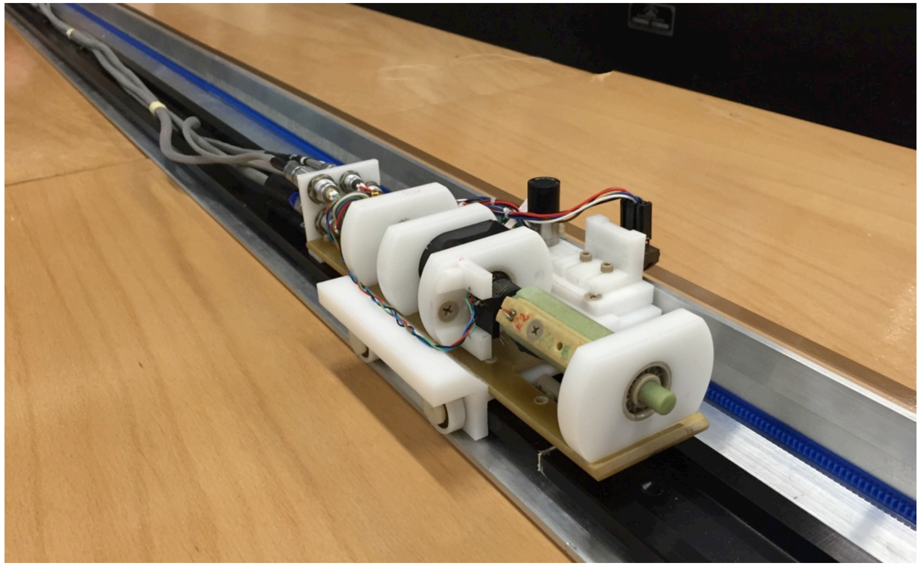
$$\overline{\phi}_\mathrm{m}(x,y) := \int_{-z_0}^{z_0} \phi_\mathrm{m}(x,y,z)\mathrm{d}z.$$

$$\begin{aligned}
\frac{\partial^2 \overline{\phi}_\mathrm{m}(x,y)}{\partial x^2} + \frac{\partial^2 \overline{\phi}_\mathrm{m}(x,y)}{\partial y^2} &= \int_{-z_0}^{z_0} \left(\frac{\partial^2 \phi_\mathrm{m}}{\partial x^2} + \frac{\partial^2 \phi_\mathrm{m}}{\partial y^2} \right) \mathrm{d}z \\
&= \int_{-z_0}^{z_0} \left(-\frac{\partial^2 \phi_\mathrm{m}}{\partial z^2} \right) \mathrm{d}z = - \left. \frac{\partial \phi_\mathrm{m}}{\partial z} \right|_{-z_0}^{z_0} \\
&= H_z(-z_0) - H_z(z_0) \overset{!}{=} 0\,.
\end{aligned}$$

The 2D scaling laws hold for the integrated harmonics

Rotating Coil Mapper (alias Toy Train)

Experimental characterization (magnetic compatibility, electrical interference, rotation quality) of a rotating coil transducer for local multipole scanning



Prototype transport system

Pseudo-Multipoles (Fourier Bessel Series)

$$\phi_{\mathrm{m}}(r,\varphi,z) = \left\{ \begin{array}{c} \cos n\varphi \\ \sin n\varphi \end{array} \right\} I_n(pr) \left\{ \begin{array}{c} \cos pz \\ \sin pz \end{array} \right\}$$

$$I_n(pr) = \sum_{k=0}^{\infty} \frac{1}{k!\,\Gamma(k+n+1)} \left(\frac{pr}{2}\right)^{n+2k}$$

$$\phi_{\mathrm{m}} = \sum_{k=0}^{\infty} \sum_{n=1}^{\infty} r^{n+2k} \left(\mathcal{C}_{n+2k,n}(z) \sin n\varphi + \mathcal{D}_{n+2k,n}(z) \cos n\varphi\right)$$

$$
\begin{aligned}
&\frac{1}{r}\frac{\partial}{\partial r}\Big\{\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}(n+2k)r^{n+2k}\left(\mathcal{C}_{n+2k,n}(z)\sin n\varphi+\mathcal{D}_{n+2k,n}(z)\cos n\varphi\right)\Big\}\\
&\quad-\frac{1}{r^2}\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}n^2\,r^{n+2k}\left(\mathcal{C}_{n+2k,n}(z)\sin n\varphi+\mathcal{D}_{n+2k,n}(z)\cos n\varphi\right)\\
&\quad+\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}r^{n+2k}\left(\mathcal{C}^{(2)}_{n+2k,n}(z)\sin n\varphi+\mathcal{D}^{(2)}_{n+2k,n}(z)\cos n\varphi\right)\\
=&\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}(n+2k)^2\,r^{n+2k-2}\left(\mathcal{C}_{n+2k,n}(z)\sin n\varphi+\mathcal{D}_{n+2k,n}(z)\cos n\varphi\right)\\
&\quad-\sum_{k=0}^{\infty}\sum_{n=1}^{\infty}n^2\,r^{n+2k-2}\left(\mathcal{C}_{n+2k,n}(z)\sin n\varphi+\mathcal{D}_{n+2k,n}(z)\cos n\varphi\right)\\
&\quad+\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}r^{n+2k-2}\left(\mathcal{C}^{(2)}_{n+2k-2,n}(z)\sin n\varphi+\mathcal{D}^{(2)}_{n+2k-2,n}(z)\cos n\varphi\right)\\
=&\;0\,, \qquad (
\end{aligned}
$$

Recursion for Cn,n

$$\mathcal{C}_{n+2k,n}(z)\left((n+2k)^2-n^2\right)+\mathcal{C}^{(2)}_{n+2k-2,n}(z)=0\,,$$

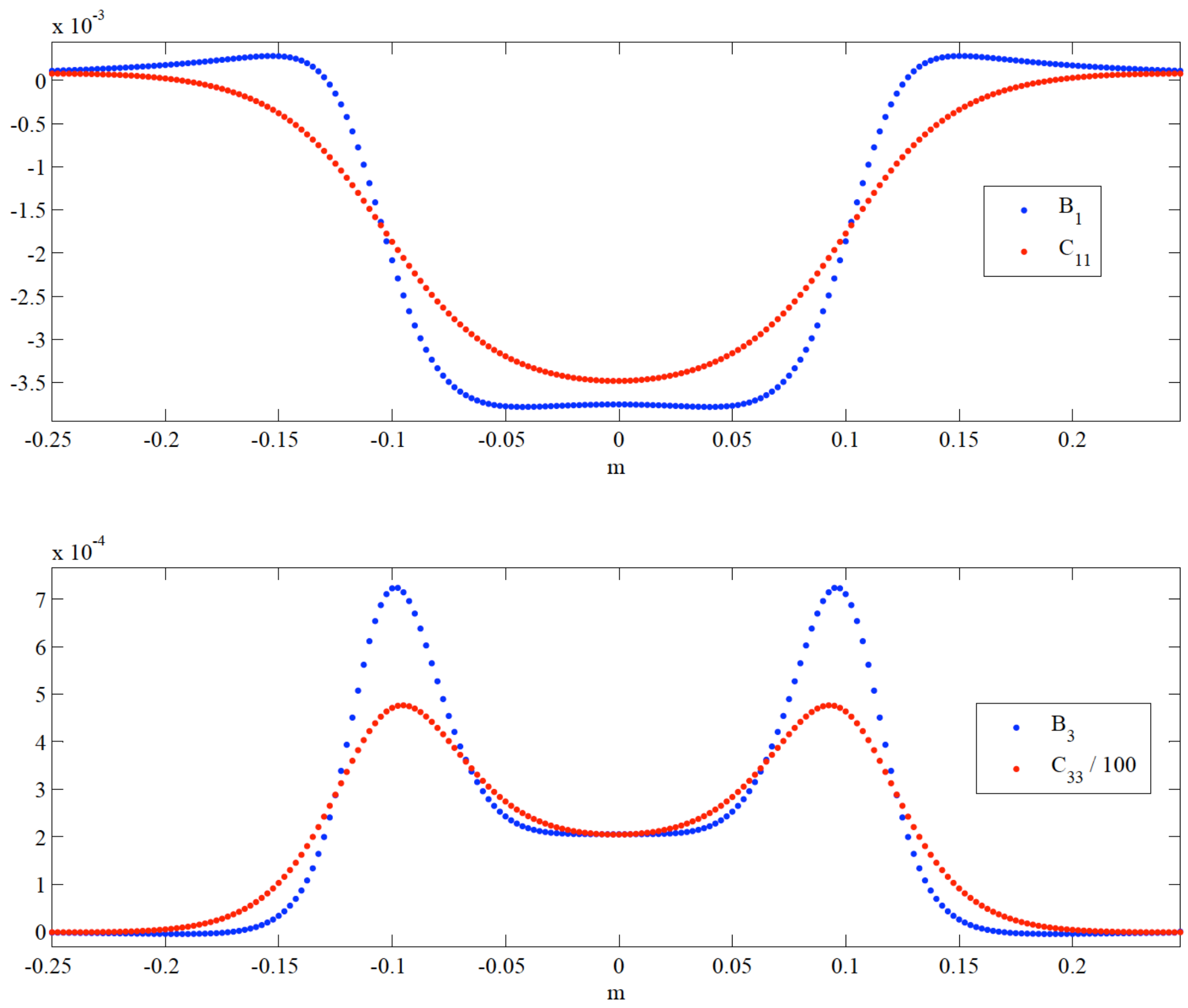
$$\mathcal{D}_{n+2k,n}(z)\left((n+2k)^2-n^2\right)+\mathcal{D}^{(2)}_{n+2k-2,n}(z)=0\,.$$

$$\mathcal{C}_{n+2k,n}(z)=\frac{1}{\prod_{m=1}^{k}(n^2-(n+2m)^2)}\,\mathcal{C}^{(2k)}_{n,n}(z)\,,$$

$$\phi_{\mathrm{m}} = \sum_{n=1}^{\infty}\left\{\sum_{k=0}^{\infty}\frac{1}{\prod_{m=1}^{k}(n^2-(n+2m)^2)}\,\mathcal{C}_{n,n}^{(2k)}(z)\right\} r^n \sin n\varphi$$
$$+ \sum_{n=1}^{\infty}\left\{\sum_{k=0}^{\infty}\frac{1}{\prod_{m=1}^{k}(n^2-(n+2m)^2)}\,\mathcal{D}_{n,n}^{(2k)}(z)\right\} r^n \cos n\varphi\,,$$

$$\phi_{\mathrm{m}} = \sum_{n=1}^{\infty}\left\{\mathcal{C}_{n,n}(z) - \frac{\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)}r^2\right.$$
$$\left. + \frac{\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r^4 - \frac{\mathcal{C}_{n,n}^{(6)}(z)}{384(n+1)(n+2)(n+3)}r^6 + \ldots\right\} r^n \sin n\varphi$$
$$+ \sum_{n=1}^{\infty}\left\{\mathcal{D}_{n,n}(z) - \frac{\mathcal{D}_{n,n}^{(2)}(z)}{4(n+1)}r^2\right.$$
$$\left. + \frac{\mathcal{D}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r^4 - \frac{\mathcal{D}_{n,n}^{(6)}(z)}{384(n+1)(n+2)(n+3)}r^6 + \ldots\right\} r^n \cos n\varphi\,,$$

The Leading Term is NOT the Measured One



Field Components from Pseudo-Multipoles

$$\phi_{\mathrm{m}}(r,\varphi) = \sum_{n=1}^{\infty} r^n (\widetilde{\mathcal{C}}_n(r,z)\sin n\varphi + \widetilde{\mathcal{D}}_n(z)\cos n\varphi)\,.$$

$$B_r(r,\varphi,z) = -\mu_0 \sum_{n=1}^{\infty} r^{n-1}(\overline{\mathcal{C}}_n(r,z)\sin n\varphi + \overline{\mathcal{D}}_n(r,z)\cos n\varphi)\,,$$

$$B_\varphi(r,\varphi,z) = -\mu_0 \sum_{n=1}^{\infty} n\, r^{n-1}(\widetilde{\mathcal{C}}_n(r,z)\cos n\varphi - \widetilde{\mathcal{D}}_n(r,z)\sin n\varphi)\,,$$

$$B_z(r,\varphi,z) = -\mu_0 \sum_{n=1}^{\infty} r^n \left(\frac{\partial \widetilde{\mathcal{C}}_n(r,z)}{\partial z}\sin n\varphi + \frac{\partial \widetilde{\mathcal{D}}_n(r,z)}{\partial z}\cos n\varphi \right)\,,$$

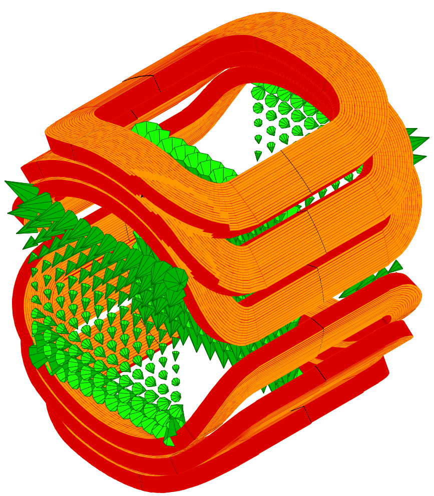
$$\overline{\mathcal{C}}_n(r,z) = n\,\mathcal{C}_{n,n}(z) - \frac{(n+2)\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)}r^2 + \frac{(n+4)\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r^4 - \dots$$

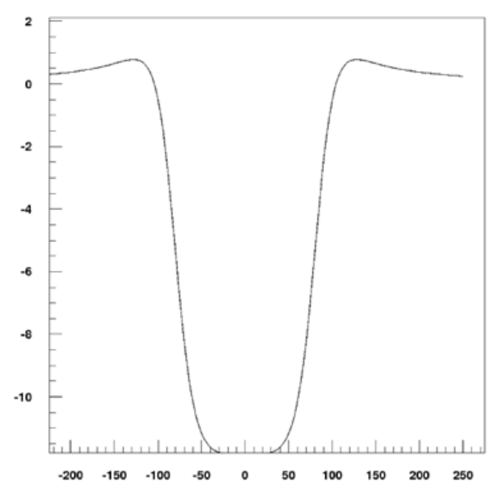
$$\widetilde{\mathcal{C}}_n(r,z) := \mathcal{C}_{n,n}(z) - \frac{\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)}r^2 + \frac{\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r^4 - \dots\,,$$

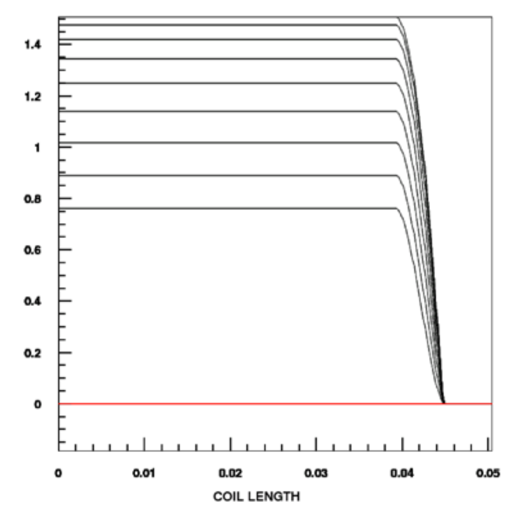
Extraction of Cn,n from Transversal Field Measurements

$$B_n(r_0,z) = -\mu_0 r_0^{n-1}\overline{\mathcal{C}}_n(r_0,z) =$$

$$-\mu_0 r_0^{n-1}\left(n\,\mathcal{C}_{n,n}(z) - \frac{(n+2)\mathcal{C}_{n,n}^{(2)}(z)}{4(n+1)}r_0^2 + \frac{(n+4)\mathcal{C}_{n,n}^{(4)}(z)}{32(n+1)(n+2)}r_0^4 - \dots\right).$$

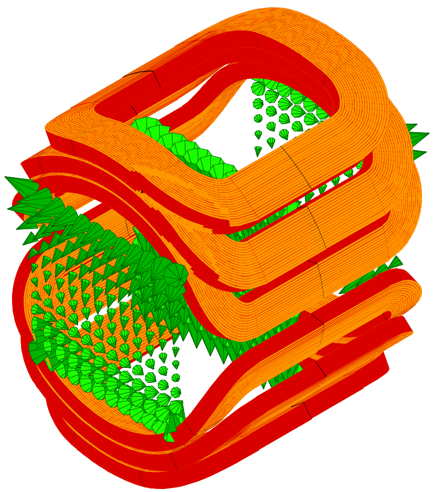


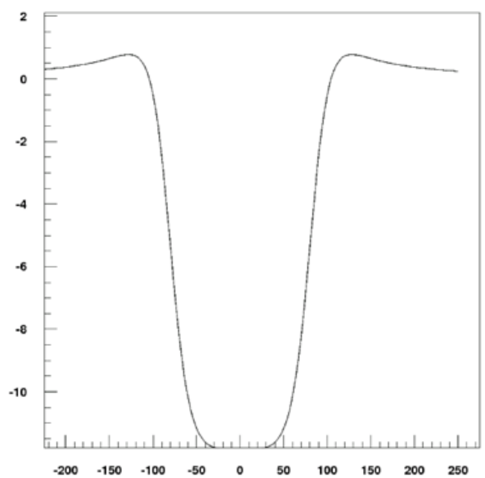


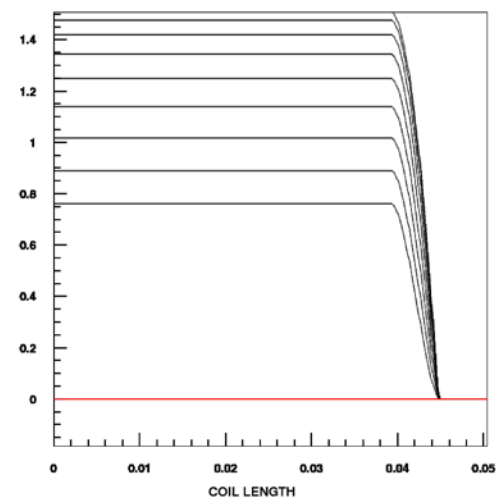


Extraction of Cn,n from Transversal Field Measurements

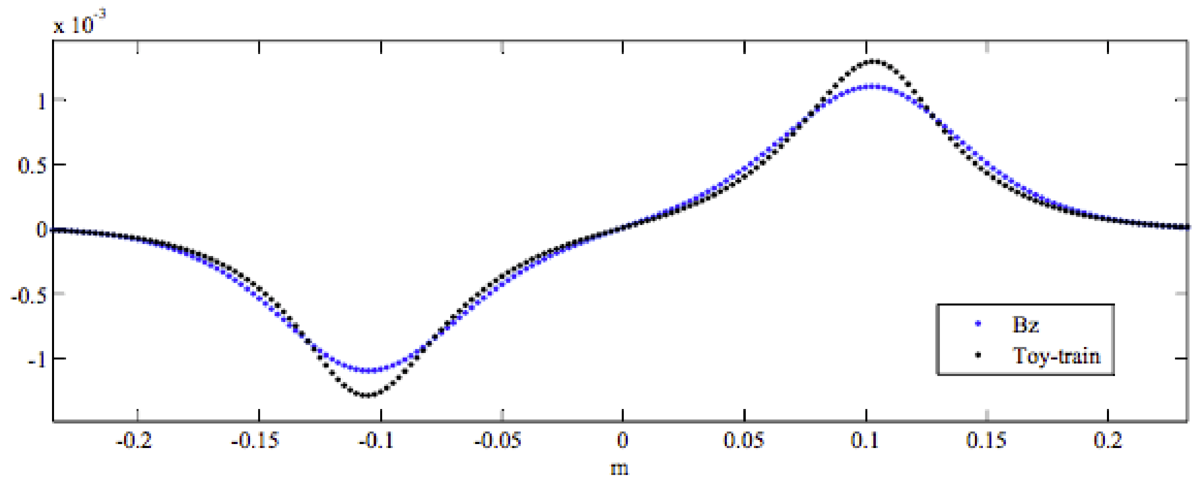
$$\mathcal{F}\{C_{n,n}(z)\} = \frac{-\mathcal{F}\{B_n(r_0,z)\}\,\mathcal{F}\{K_n(r_0,z)\}}{\mu_0\, r_0^{n-1}\left(n - \frac{(n+2)(i\omega)^2}{4(n+1)} r_0^2 + \frac{(n+4)(i\omega)^4}{32(n+1)(n+2)} r_0^4 - \dots\right)}.$$

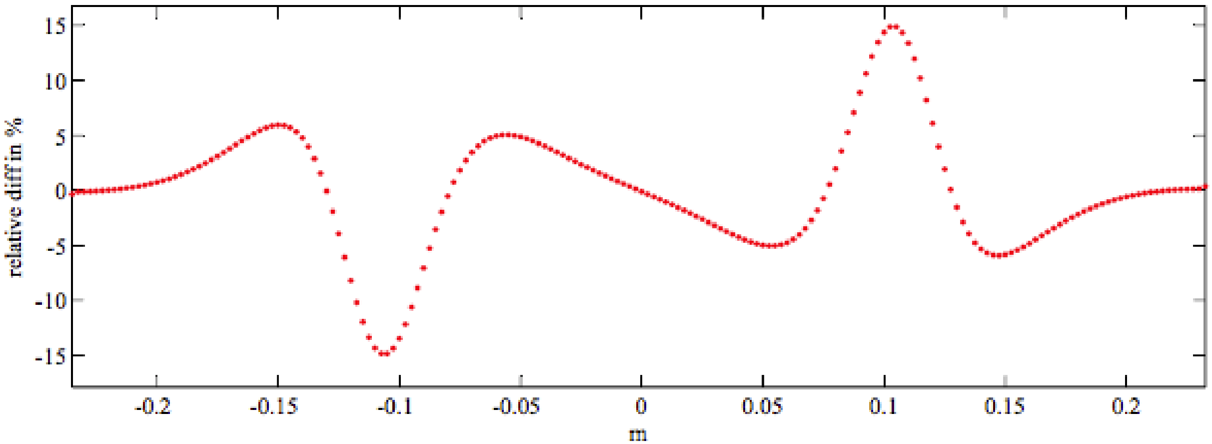




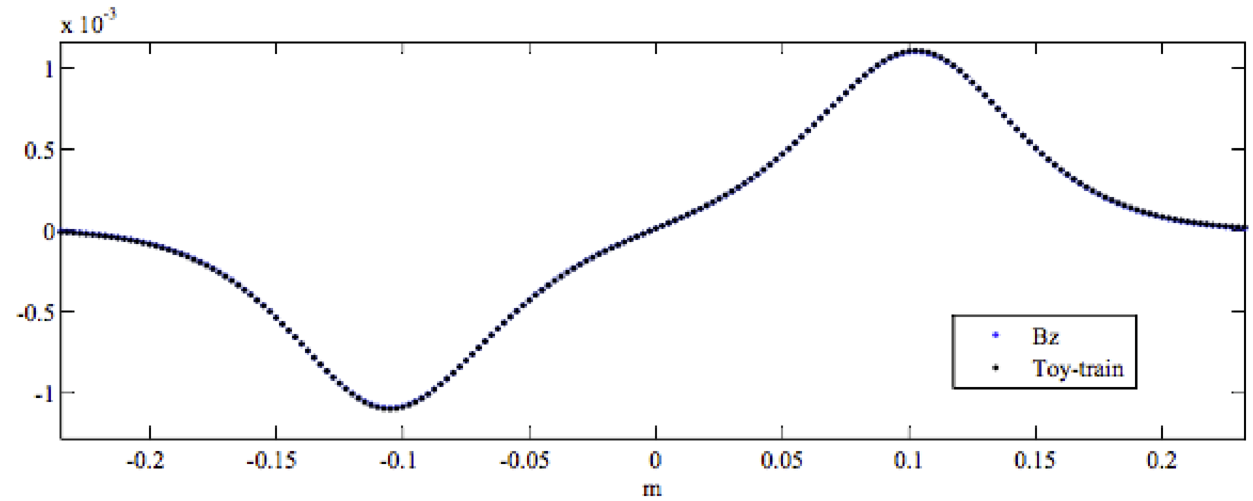


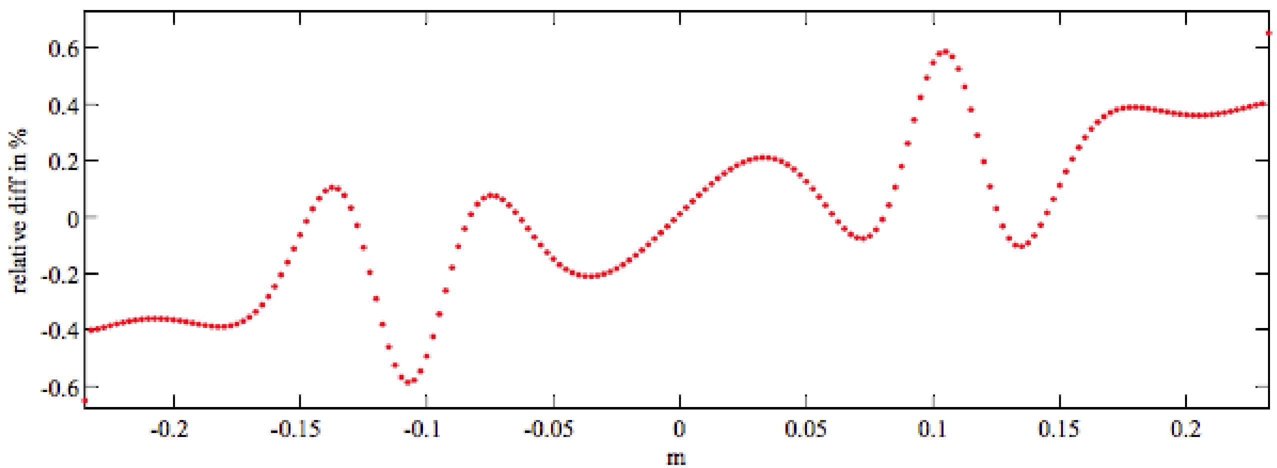
Longitudinal Field Reconstruction (second order pseudo-mulipole)



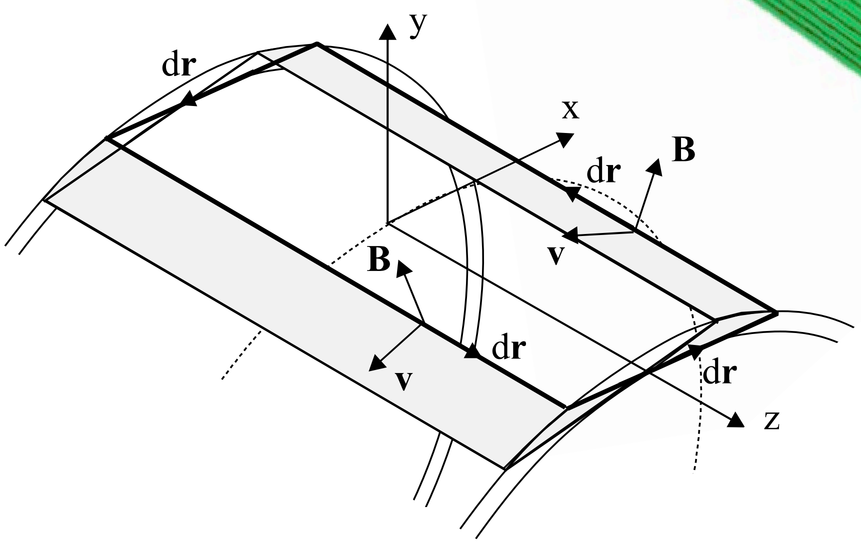


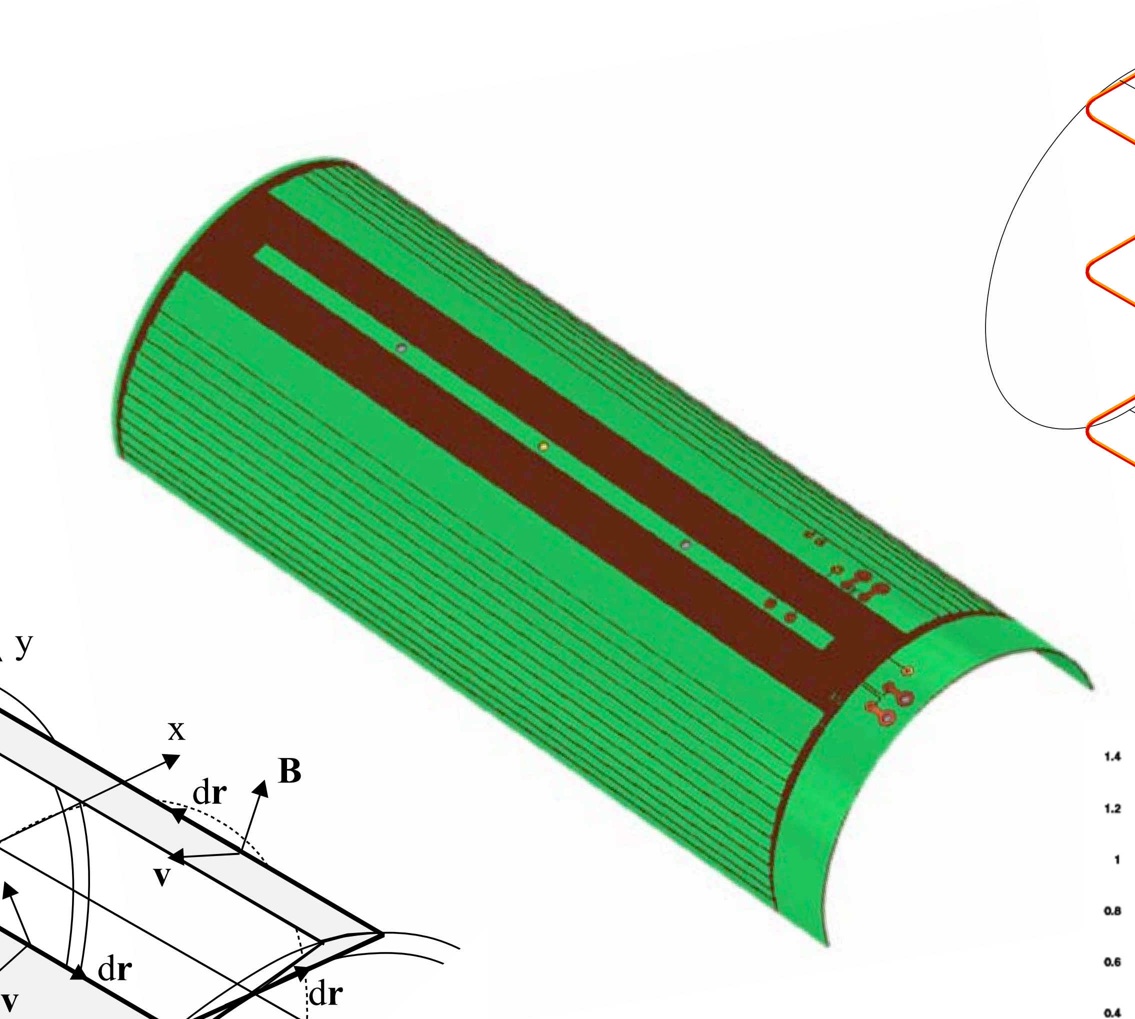
Longitudinal Field Reconstruction (6th order speudo-mulipole)

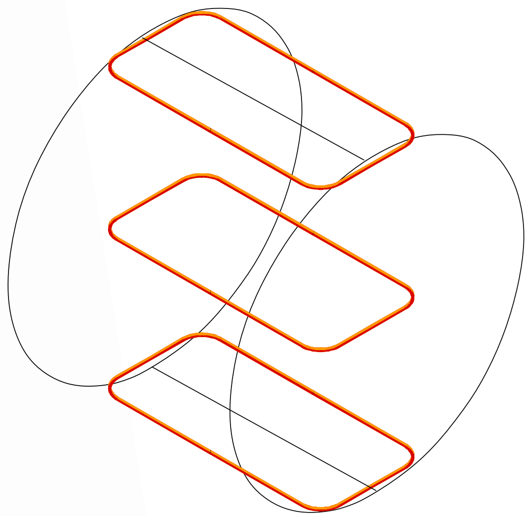


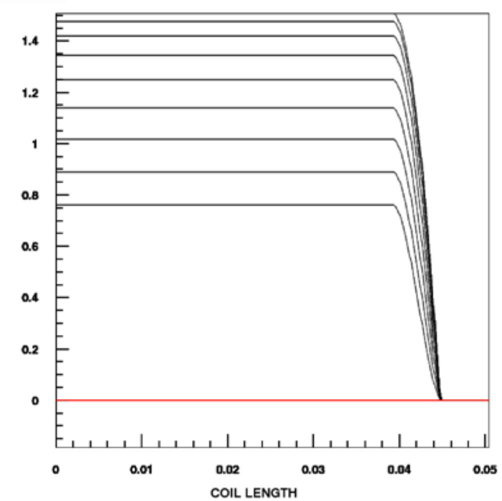


Search Coils Must be Saddle-Shaped

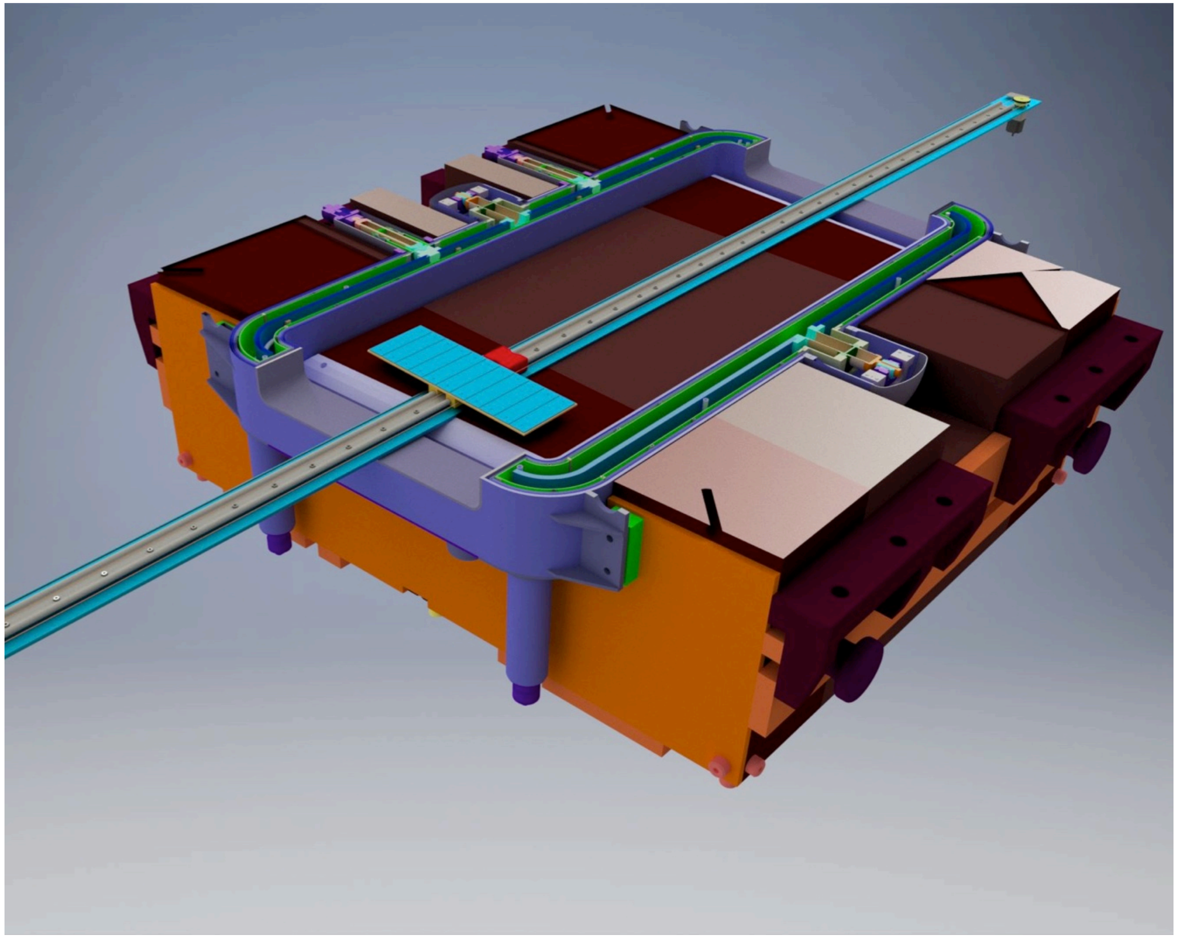


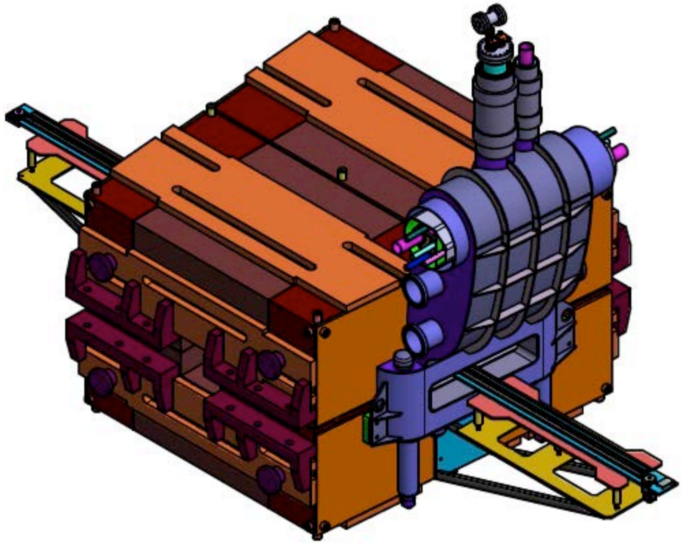




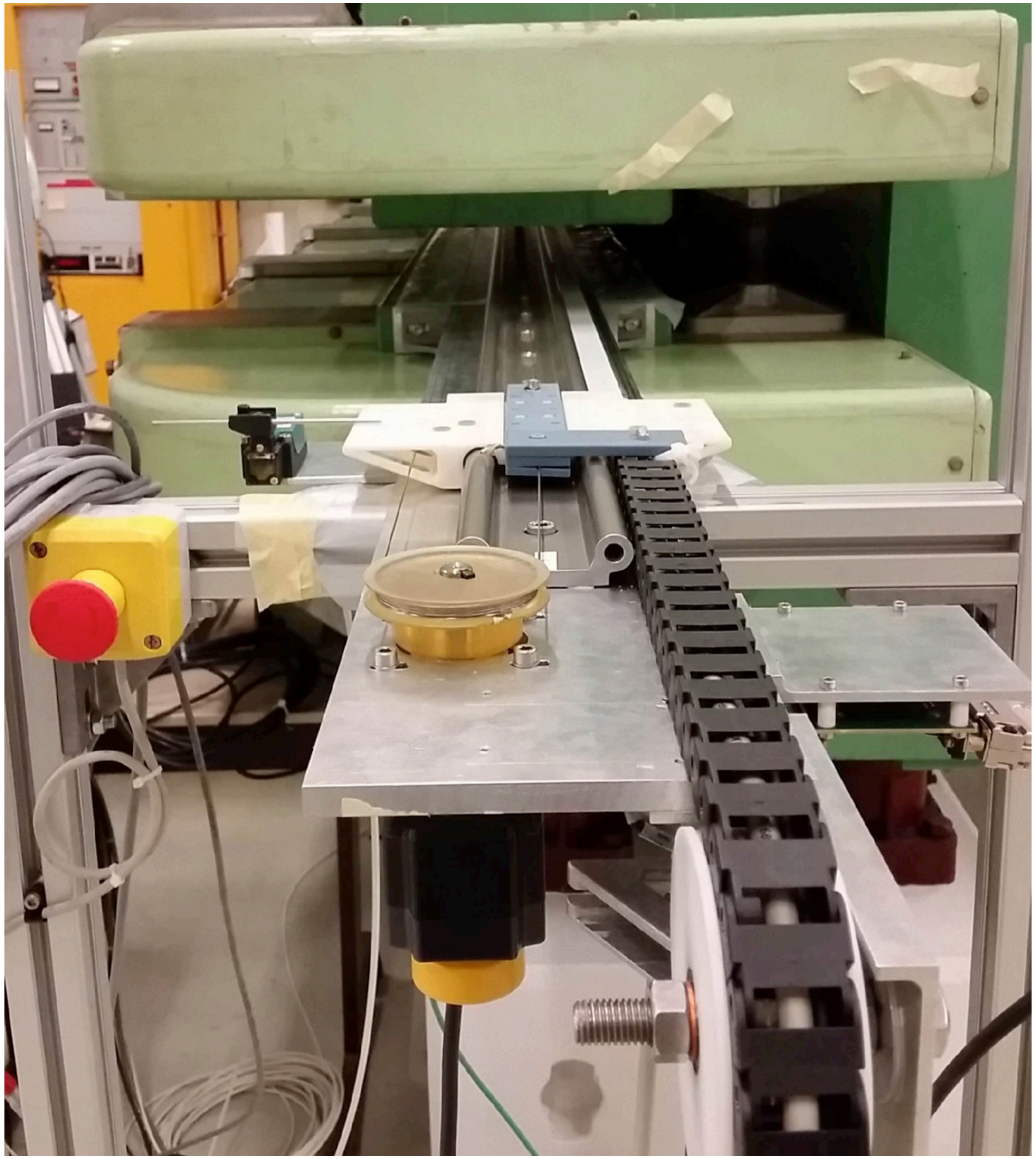


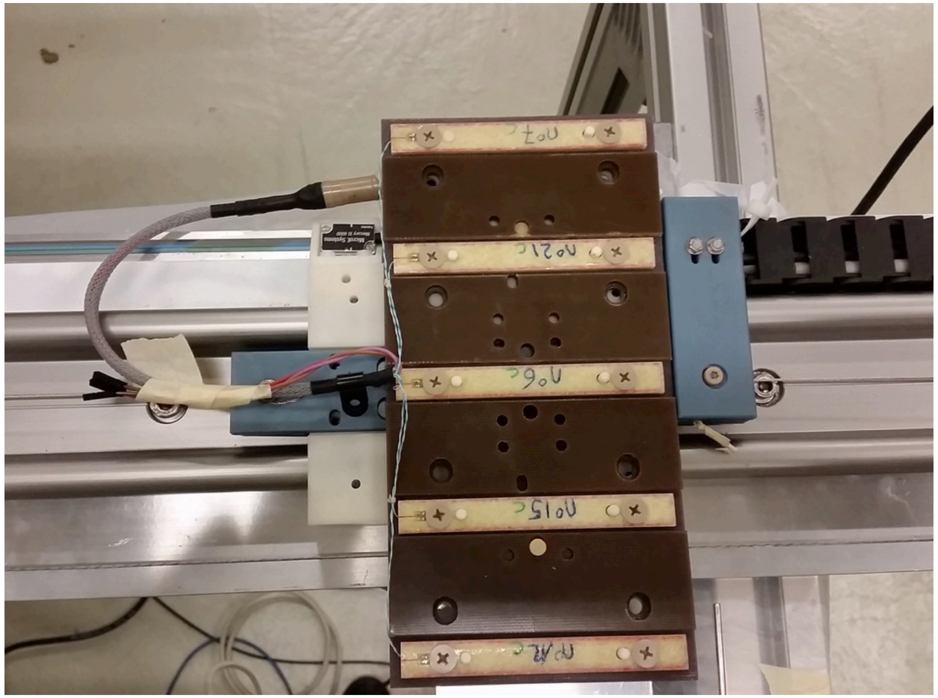
Translating Fluxmeter



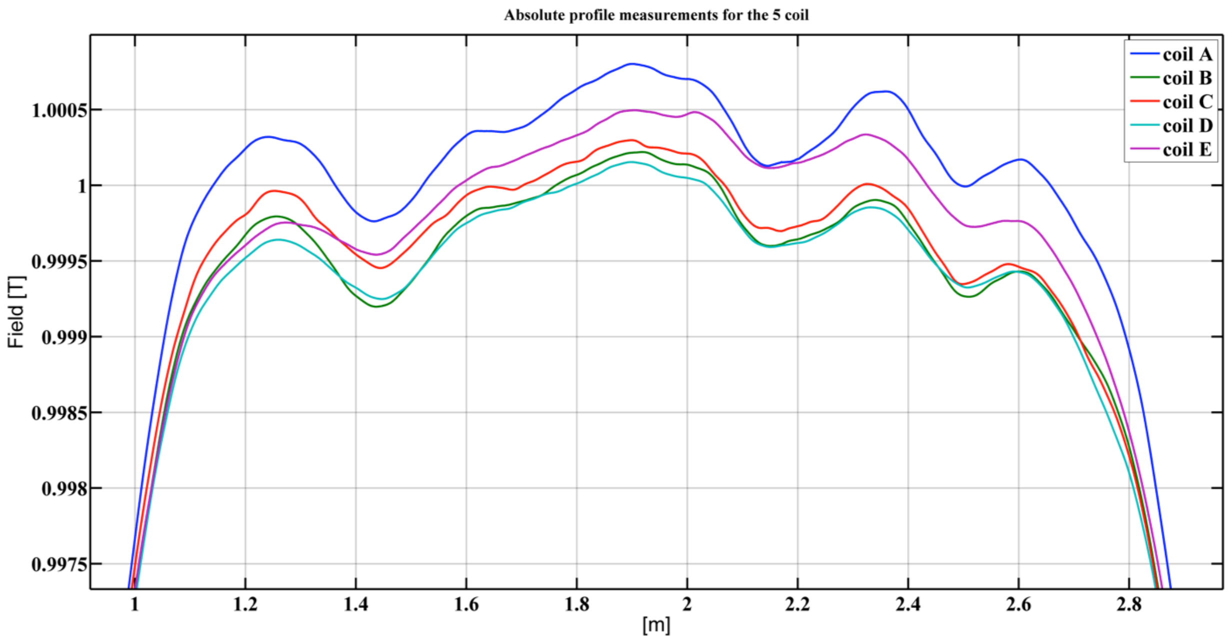


The Translating Fluxmeter (Prototype)



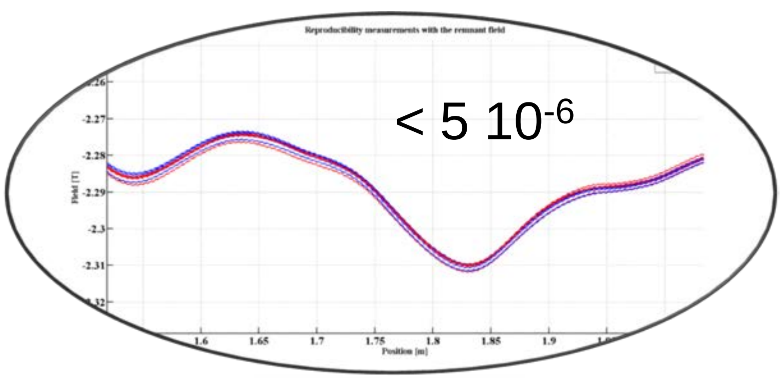


Longitudinal Profile Measurements

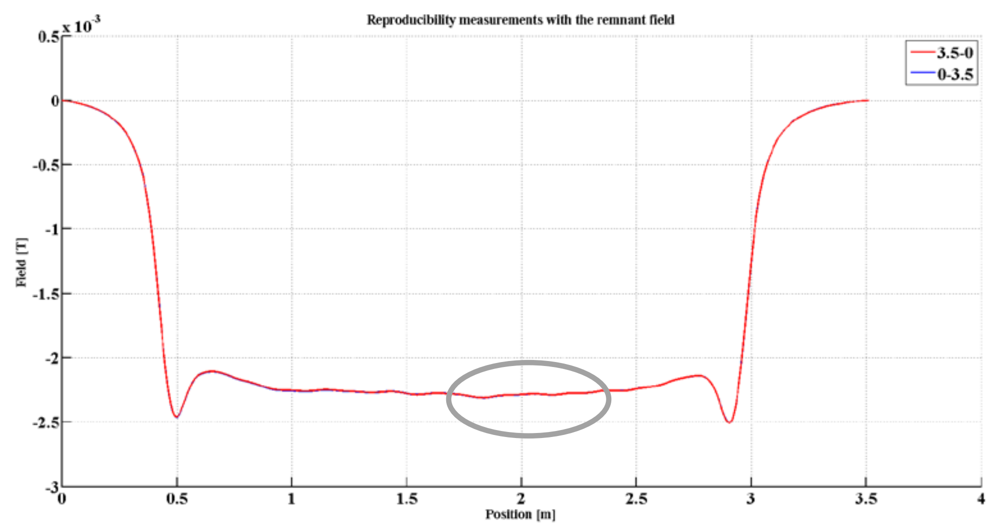


Absolute for the 5 tracks

Go return



< 5 10^{-6}



Mid-Plane Field

$$\frac{-1}{\mu_0} B_y(x, y=0, z) \approx$$

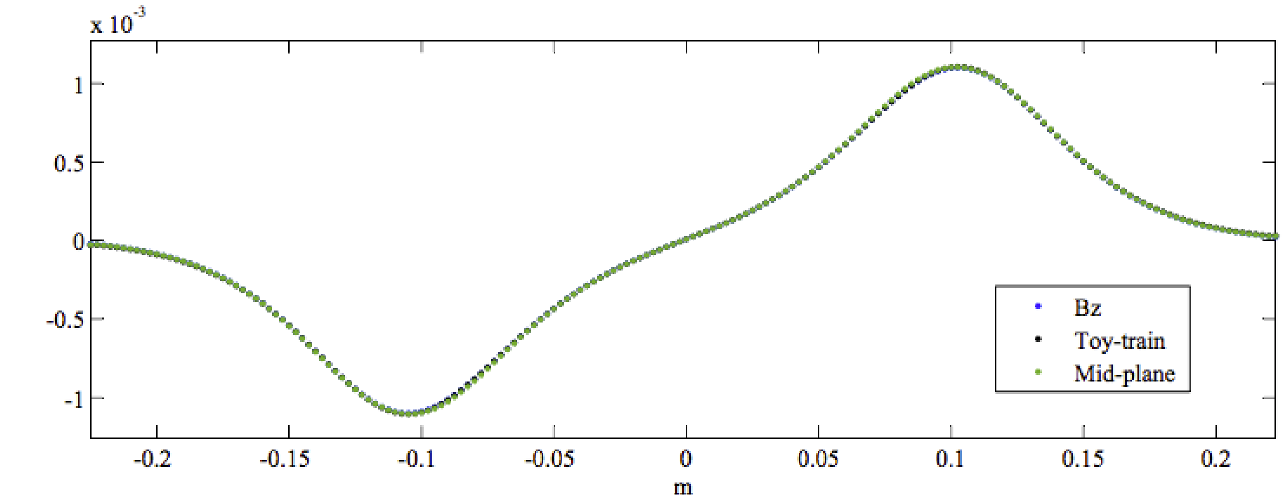
$$\mathcal{C}_{1,1}(z) \quad - \frac{\mathcal{C}_{1,1}^{(2)}(z)}{8} x^2 \quad + \frac{\mathcal{C}_{1,1}^{(4)}(z)}{192} x^4 \quad - \frac{\mathcal{C}_{1,1}^{(6)}(z)}{9216} x^6$$

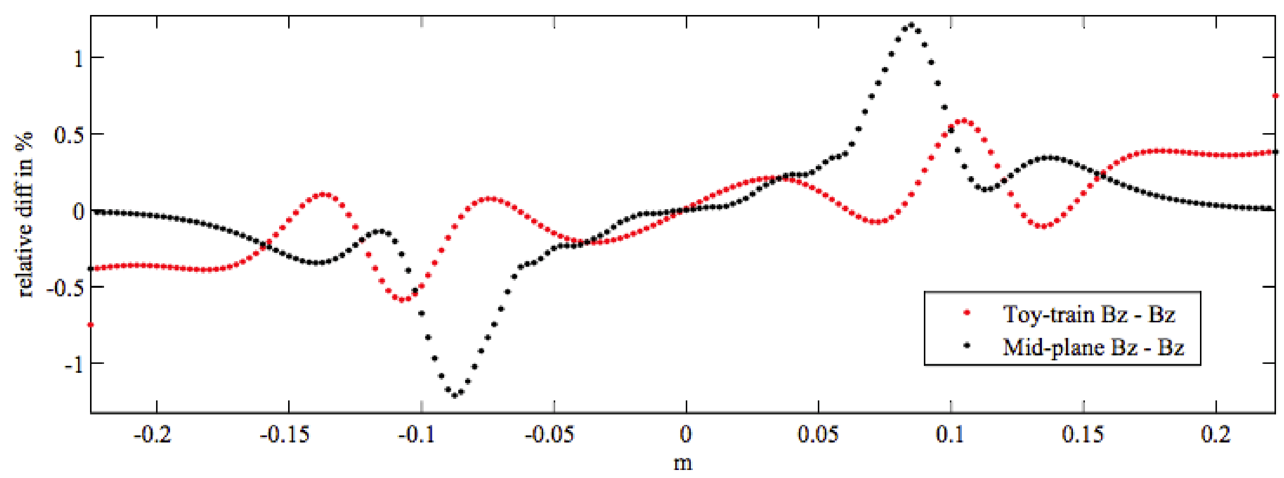
$$+ 3\,\mathcal{C}_{3,3}(z)\,x^2 - \frac{3\,\mathcal{C}_{3,3}^{(2)}(z)}{16} x^4 + \frac{3\,\mathcal{C}_{3,3}^{(4)}(z)}{640} x^6$$

$$+ 5\,\mathcal{C}_{5,5}(z)\,x^4 - \frac{5\,\mathcal{C}_{5,5}^{(2)}(z)}{24} x^6$$

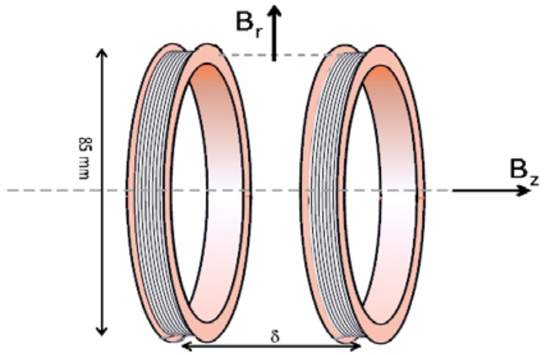
$$+ 7\,\mathcal{C}_{7,7}(z)\,x^6$$

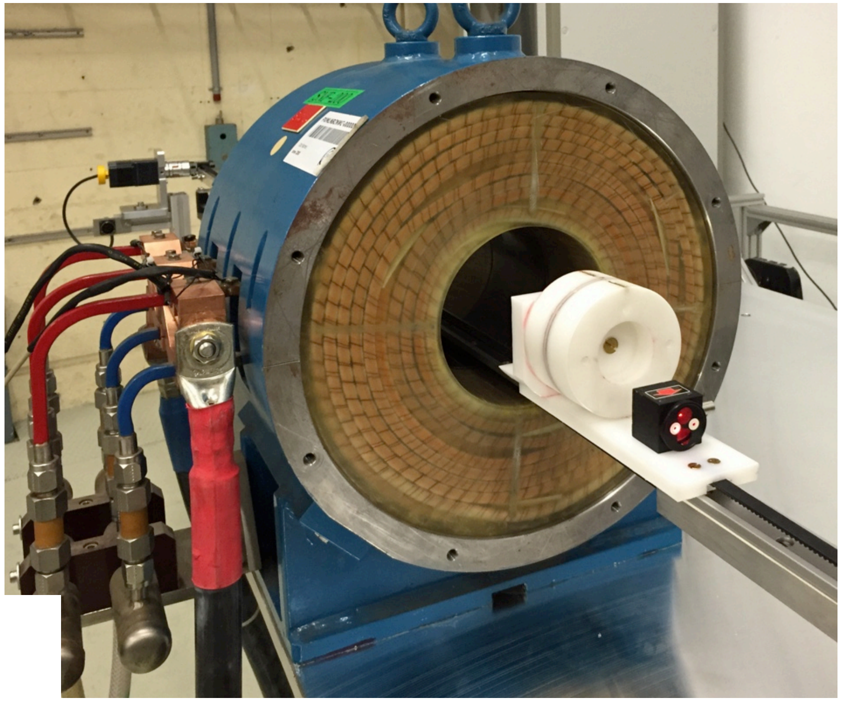
Longitudinal Field Reconstruction (comp. Toy Train, Moving Fluxmeter)





Solenoid Field Transducer





$$B_z(r,z) = -\mu_0 \left(\mathcal{C}_{0,0}^{(1)}(z) - \frac{\mathcal{C}_{0,0}^{(3)}(z)}{4} r^2 + \frac{\mathcal{C}_{0,0}^{(5)}(z)}{64} r^4 \right)$$

$$B_r(r,z) = -\mu_0 \left(-\frac{\mathcal{C}_{0,0}^{(2)}(z)}{2} r + \frac{\mathcal{C}_{0,0}^{(4)}(z)}{16} r^3 \right)$$

$$\mathcal{F}\{B_r(r_0,z)\} = -\mu_0\, \mathcal{F}\{\mathcal{C}_{0,0}(z)\} \left(-\frac{(i\omega)^2}{2} r_0 + \frac{(i\omega)^4}{16} r_0^3 \right)$$

Technical Problems to Overcome

- ➔ Iso-perimetric (saddle-shaped) search coils
- ➔ Different analogue bucking scheme as radius scaling is no more valid
- ➔ No feed-down correction for axis misaligment
- ➔ Induced voltages in connectors, wiring
- ➔ Accuracy in longitudinal coil positioning
- ➔ Interference with martensitic/conducting elements in the transducer
- ➔ Effect of measurement errors on deconvolution and field reconstruction