Measurement of the warm Current Center Line of ITER Toroidal Field coils

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2. Theory
3. Instrumentation
4. Eddy current compensation
5. Dual Pancake Prototype test & results
6. Winding Pack 11 test & results
7. Conclusions

Introduction

ITER

- Largest-ever thermonuclear fusion experiment and international scientific collaboration
- Currently in construction at Cadarache (Marseille)

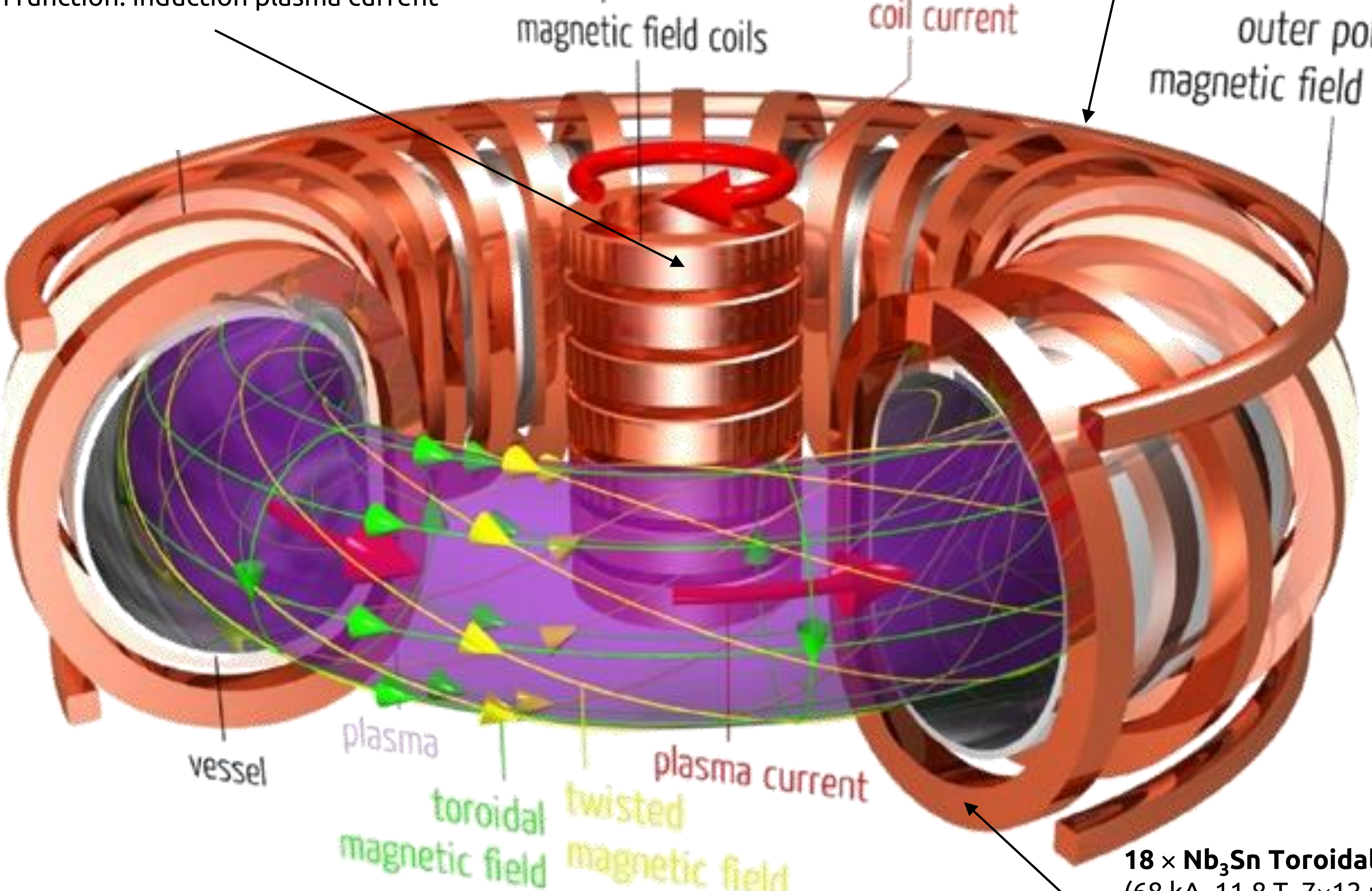
ITER Magnet System

Nb_3Sn Central Solenoid
(46 kA, 13.0 T, Ø4.3×18 m, 2163 ton, 7 GJ)

Main function: induction plasma current

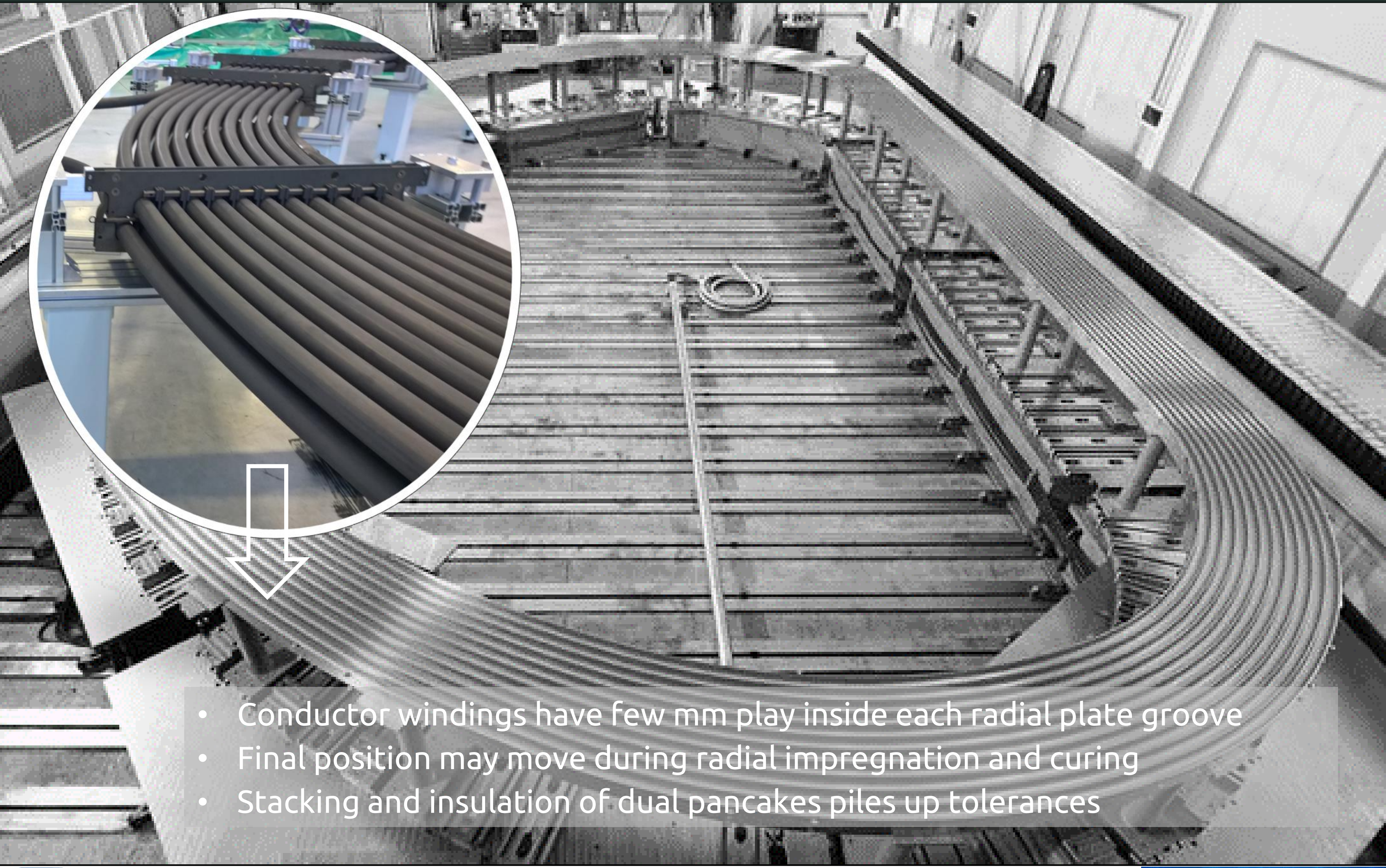
6 × NbTi Poloidal Field coils
(55 kA, 6.0 T, 298 ton, 4 GJ)

Main function: plasma confinement



18 × Nb_3Sn Toroidal Field coils
(68 kA, 11.8 T, 7×12 m, 310 ton, 41 GJ)

Main function: plasma stabilization



- Conductor windings have few mm play inside each radial plate groove
- Final position may move during radial impregnation and curing
- Stacking and insulation of dual pancakes piles up tolerances

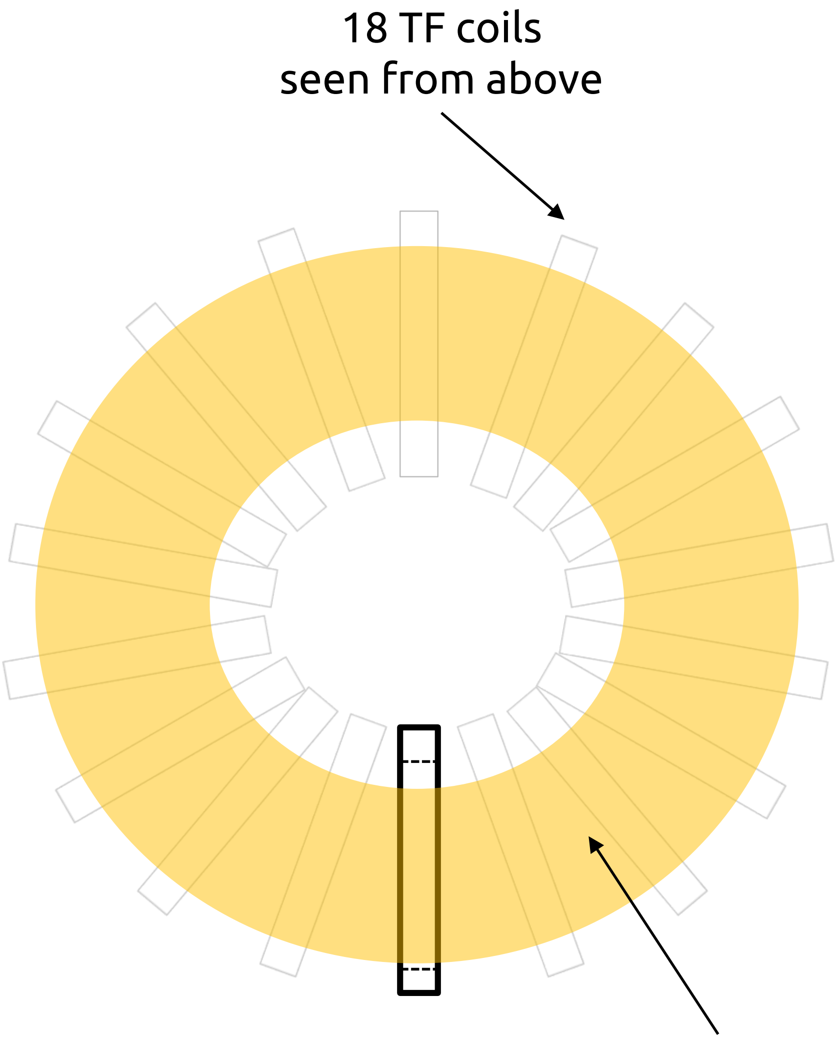
Objectives

- Impact of systematic error fields: magnetic islands leading to locked modes and disruptions, deviation of field lines leading to **localised heat deposition**
- Correction coil limit: $\mathbf{1.5 \cdot 10^{-4}}$
- At the moment, no warm or cold magnetic measurements of the assembled TF coil system is foreseen
- Current baseline: measure magnetically at RT every individual TF coil in order to:

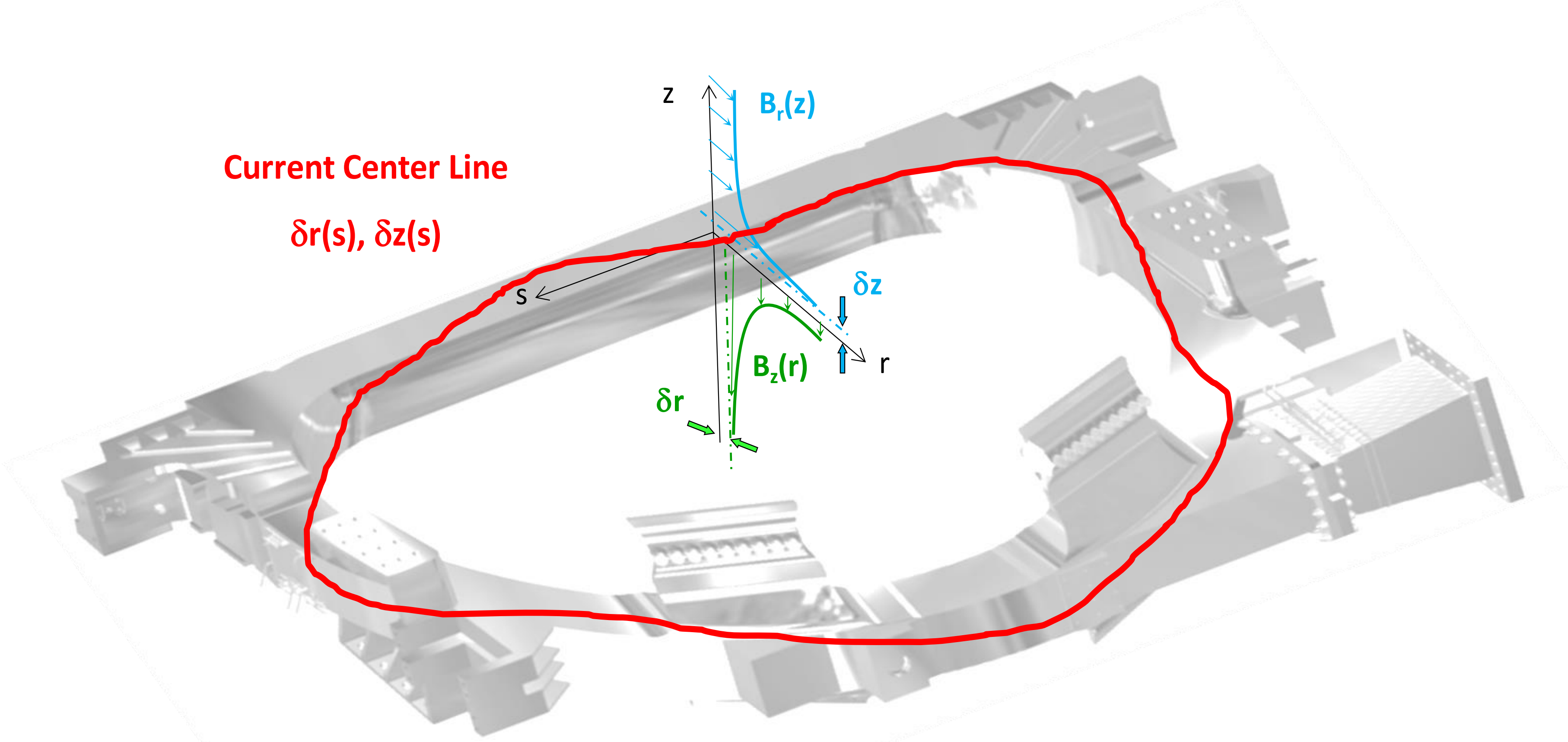
  1) optimize insertion of each Winding Pack in its casing

  2) superpose the field maps taking into account thermal contractions, mechanical deformation and tolerances

Option 1: field mapping



- Measure for every coil the map of the field it generates over the whole plasma volume
- Absolutely impractical !!



- measure the vertical and radial field profiles at several stations along a TF coil
- reconstruct the 3D shape of the equivalent current source by best-fitting a filamentary Biot-Savart model.
- The coil is excited in AC mode and the field is measured inductively with flux loops

Theory

Inverse electromagnetic problem

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \iiint_{\mathcal{V}} \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} d\mathcal{V}'$$

Direct problem: compute the field from a known current distribution (divergence-free over the domain V)

$$\boldsymbol{J} = \frac{1}{\mu_0} \nabla \times \boldsymbol{B}$$

Inverse problem: compute the current distribution form Ampere's Law V → need field everywhere !

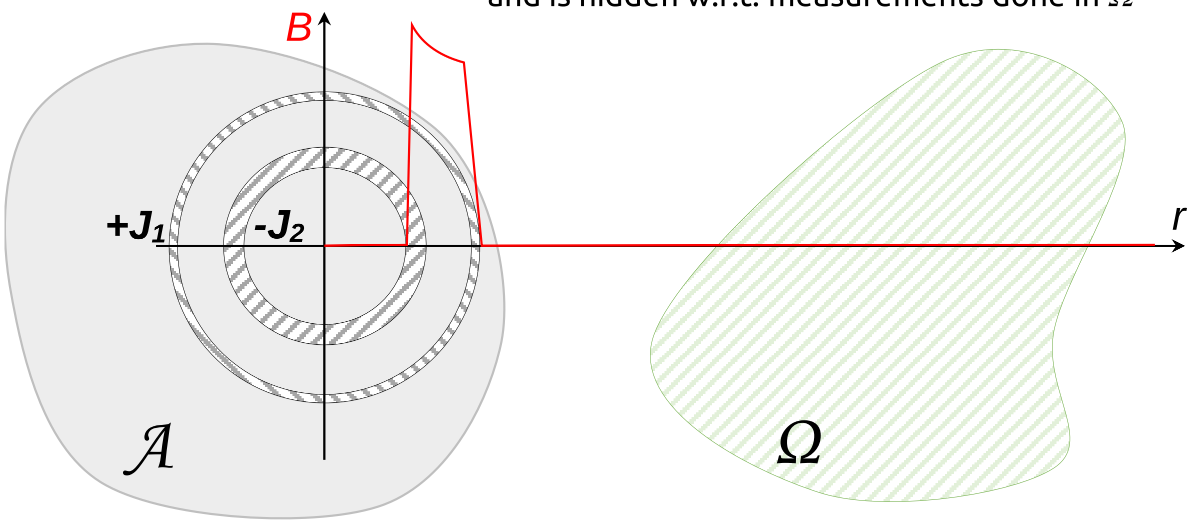
In general: the inverse problem is intrinsically ill-posed (non-unique solution)

2D example: reconstruct the current distribution in A on the basis of magnetic field measurements in Ω

add to your problem a current density in two thin concentric rings such that:

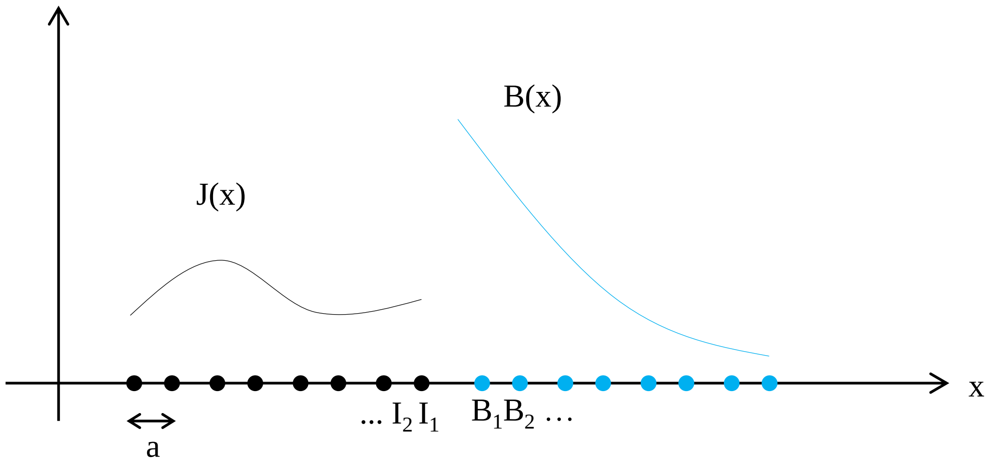
$$J_1 r_1 t_1 - J_2 r_2 t_2 = 0$$

the additional field is confined between the rings and is hidden w.r.t. measurements done in Ω



1D case study

Find the unknown linear current density profile $J_z(x)$ on the basis of exterior measurements of the magnetic flux density $B_y(x)$



$$B_{yj} = \frac{\mu_0}{2\pi}\sum_k \frac{I_k}{x_j - x_k} = \frac{\mu_0}{2\pi a}\sum_k \frac{I_k}{j+k-1}$$

$$\frac{\mu}{2\pi a} HI = B_y \qquad H = \begin{bmatrix} 1 & \frac{1}{2} & \cdots & \frac{1}{n} \\ \frac{1}{2} & \frac{1}{3} & & \frac{1}{n+1} \\ \vdots & & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n+1} & \cdots & \frac{1}{2n-1} \end{bmatrix}$$

Hilbert matrix

$$\kappa(H) = \|H\|_2 \|H^{-1}\|_2 = \frac{max\,\lambda(H)}{min\,\lambda(H)} \propto \frac{\left(1+\sqrt{2}\right)^{4n}}{\sqrt{n}}$$

$$\frac{\|\Delta I\|_2}{\|I\|_2} \leq \kappa(H)\frac{\|\Delta B\|_2}{\|B\|_2}$$

- The condition number provides an upper bound for the amplification of relative errors in the solution of linear systems
- the condition number is of the order of 10^3 already for n=2, which proves the practical impossibility of solving the problem with any reasonable level of accuracy

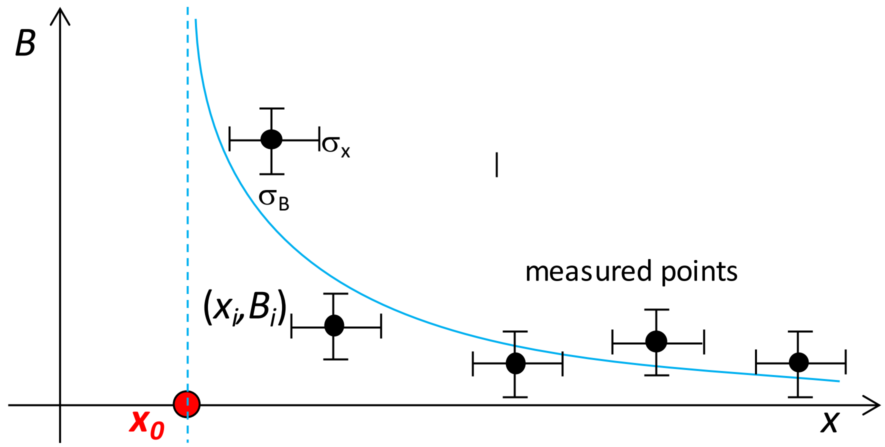
Least-squares approach

- Abandon the hope of exact solution → least-squares regularize the problem → stable solution
- 1D case study: best fit of a single-filament model to measurements with errors in both coordinates
- Total Least Squares solution with the standardized principal component method†

$$B_y(x) = \frac{\mu_0 I}{2\pi(x-a)}$$

$$a = \frac{1}{n}\sum_i x_i - \frac{\mu_0 I}{2\pi}\frac{1}{n}\sum_i \frac{1}{B_i}$$

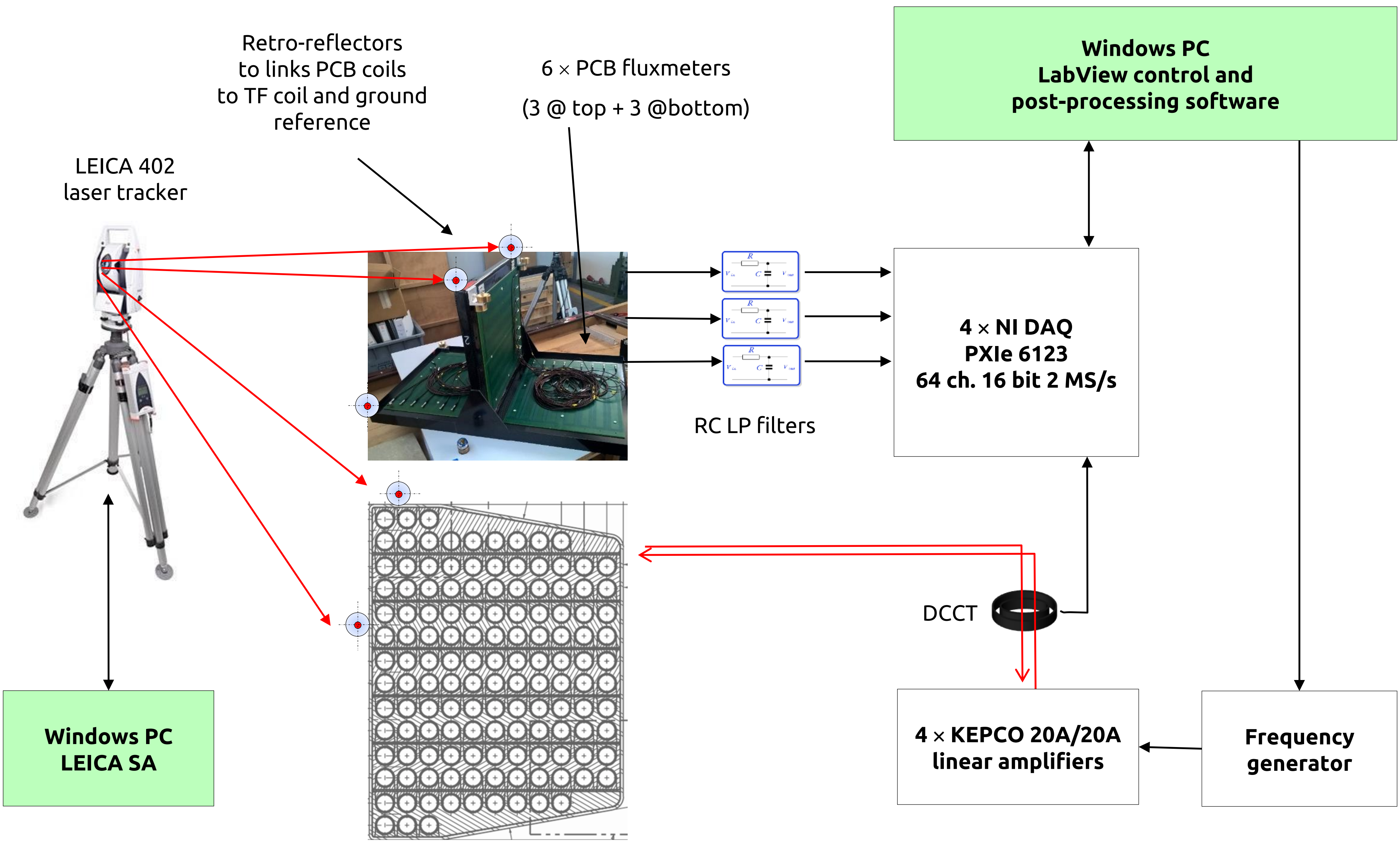
$$\frac{\sigma_a^2}{a^2} \approx \frac{1}{n}\left(\frac{\sigma_x^2}{a^2} + \frac{\sigma_B^2}{B^2}\right) + \frac{\sigma_I^2}{I^2}$$

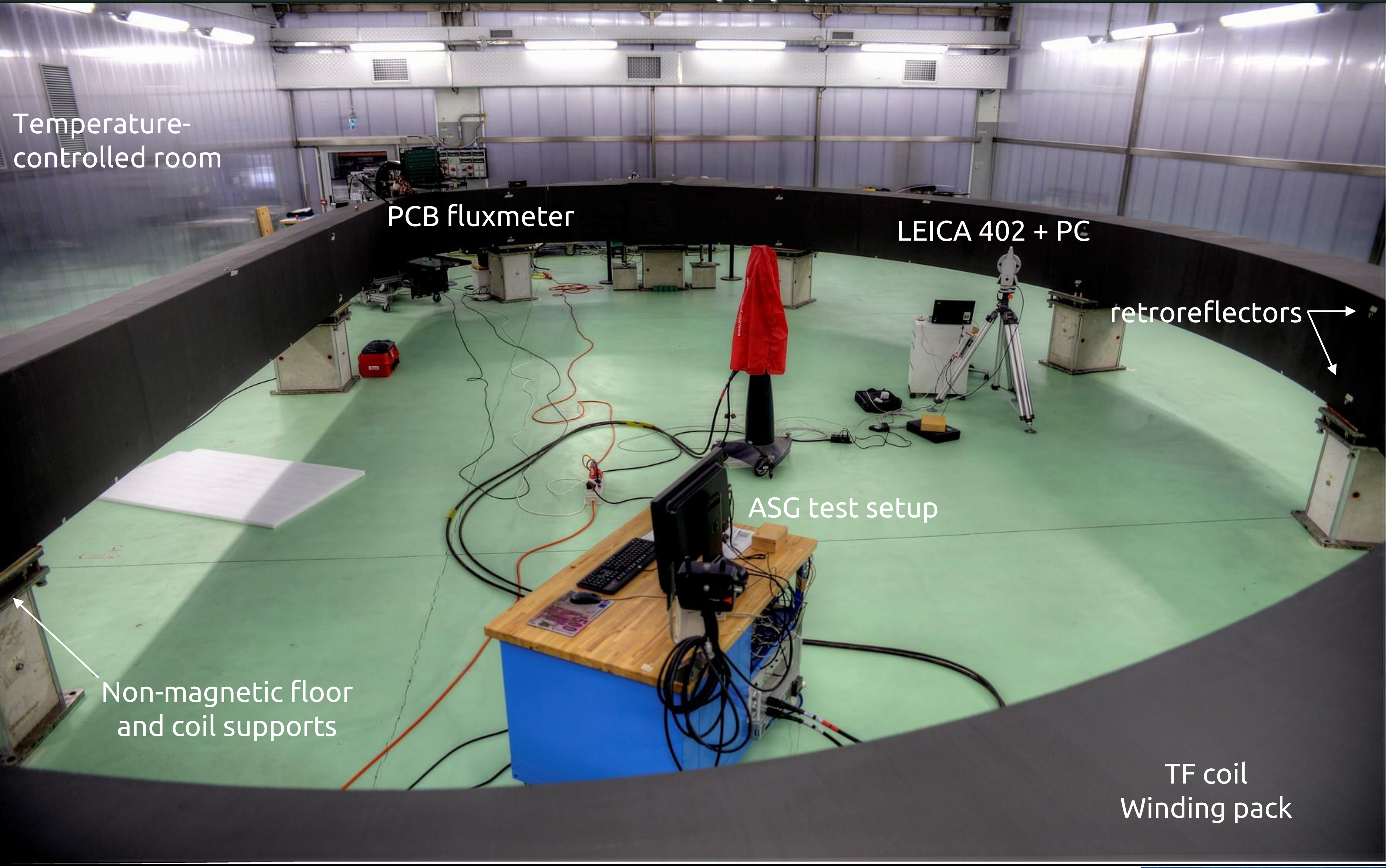


- restricting measurements to the highest possible field region is therefore very advantageous
- impact of random errors decreases with the square root of the number of measurements

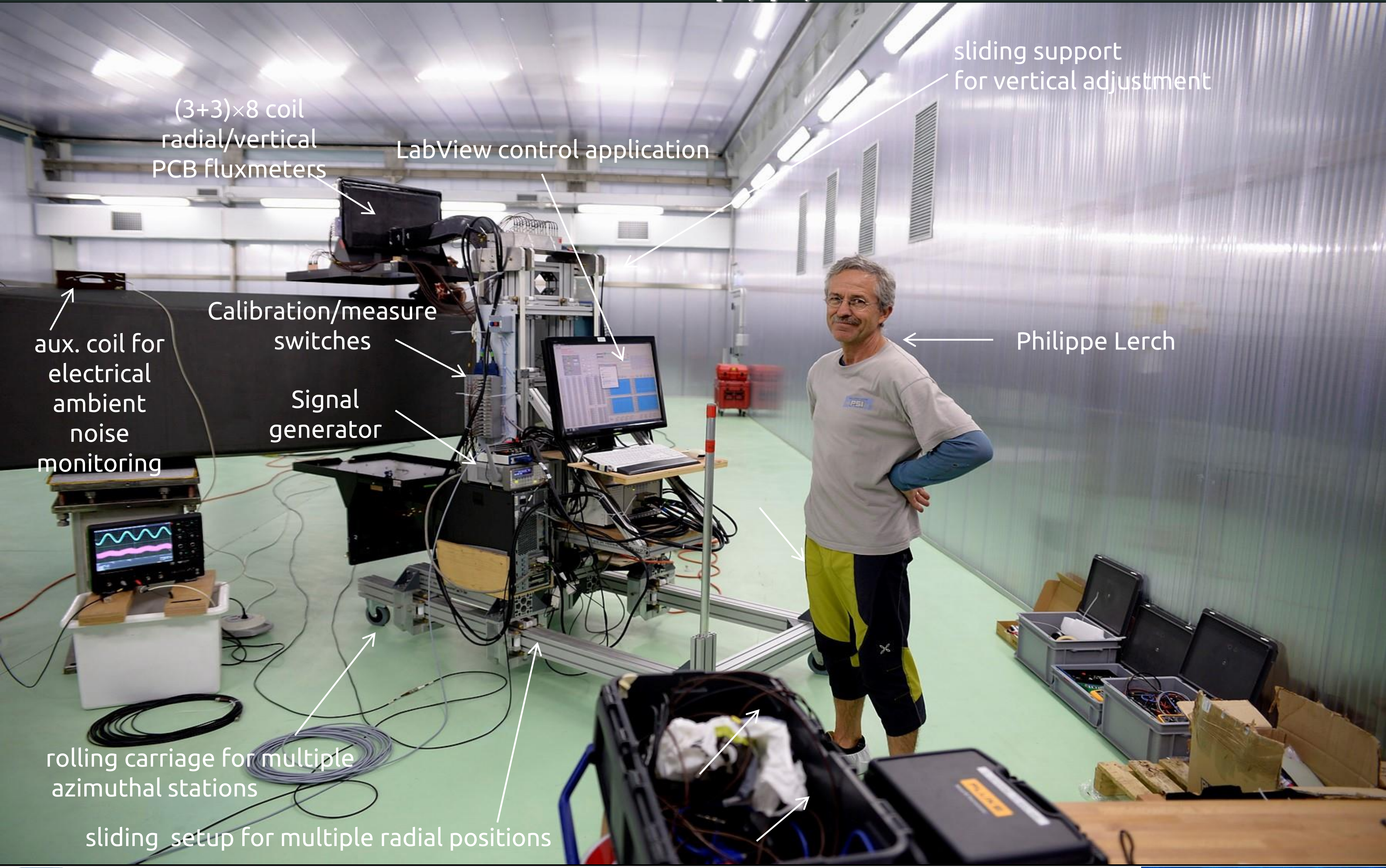
†W. Bablock, H. Passing, "Application of statistical procedures in analytical instrument testing", Journal of Automatic Chemistry, Vol. 7, No. 2, Apr-Jun 1985

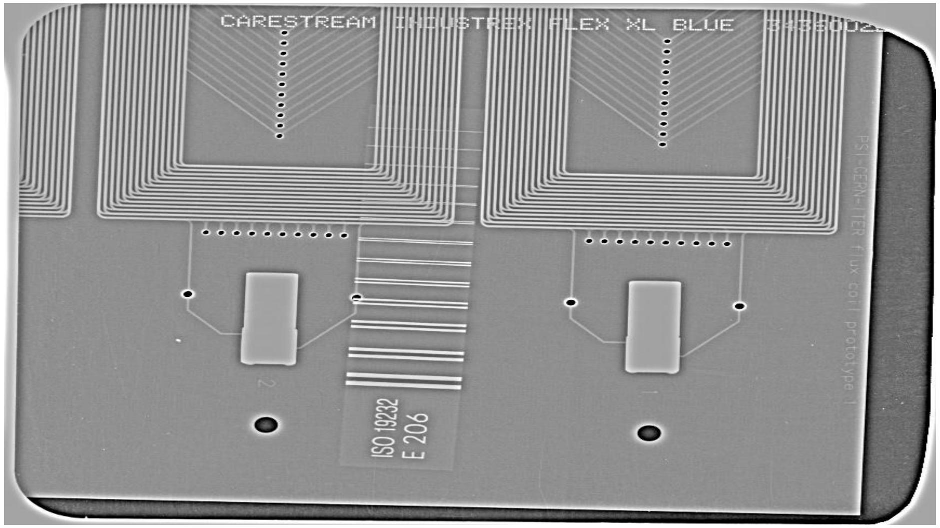
Test Setup


Retro-reflectors
to links PCB coils
to TF coil and ground
reference
6 × PCB fluxmeters
(3 @ top + 3 @bottom)
Windows PC
LabView control and
post-processing software
LEICA 402
laser tracker
RC LP filters
4 × NI DAQ
PXIe 6123
64 ch. 16 bit 2 MS/s
DCCT
Windows PC
LEICA SA
4 × KEPCO 20A/20A
linear amplifiers
Frequency
generator


Temperature-
controlled room
PCB fluxmeter
LEICA 402 + PC
retroreflectors
ASG test setup
Non-magnetic floor
and coil supports
TF coil
Winding pack

Test setup (3/4)



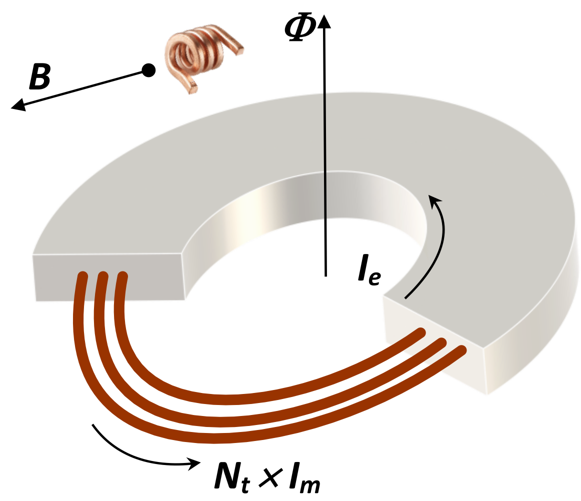


- 12 +1 PCB fluxmeters made by AD+T, Wetzikon (CH)
- 8 × 4.5 m^2 coils, 22 layers, 14 turns/layer, 412 mm × 50 mm size
- Paired in a back-to-back configuration to double the sensing surface
- Mounted on a carbon-fiber support
- Six retro-reflectors/support, three visible in any test configuration

Eddy current compensation

Lumped-circuit eddy current model

Assumptions:
- linear magnetic circuit (constant inductances)
- eddy currents couple back to the magnet circuit
- point-like, fluxmetric measurement
- magnet current I_m is known (measured)
- steady state AC
- all magnet turns have the same geometry



- Self-inductance of magnet coils: $L_m = \lambda_m \mu_0 \mu_r a N_t^2$
- Self-inductance of eddy circuit: $L_e = \lambda_e \mu_0 \mu_r a$
- Mutual inductance eddy/magnet: $L_{em} = \lambda_{em} \mu_0 \mu_r a N_t$
- Resistance of eddy circuit: $R_e = \frac{2\pi a}{A_e} \rho_e$
- Eddy time constant: $\tau_e = \frac{L_e}{R_e} = \frac{\lambda_e}{2\pi} \frac{\mu_0 \mu_r}{\rho_e} A_e$
- Aux. time constant: $\tau_{em} = \frac{L_{em}}{R_e} = \eta N_t \tau_e$
- Eddy transfer function: $k_e = \gamma \frac{k_m}{N_t}, \quad \lim_{r\to\infty} \gamma = 1$
- Geometric eddy coefficient: $\varepsilon = 1 - \gamma \frac{\lambda_{em}}{\lambda_e}$

$$\begin{cases} L_{em}\frac{dI_m}{dt} + L_e\frac{dI_e}{dt} + R_e I_e = 0 \\ B = k_m I_m + k_e I_e \\ V_c = A_c \frac{dB}{dt} \end{cases}$$

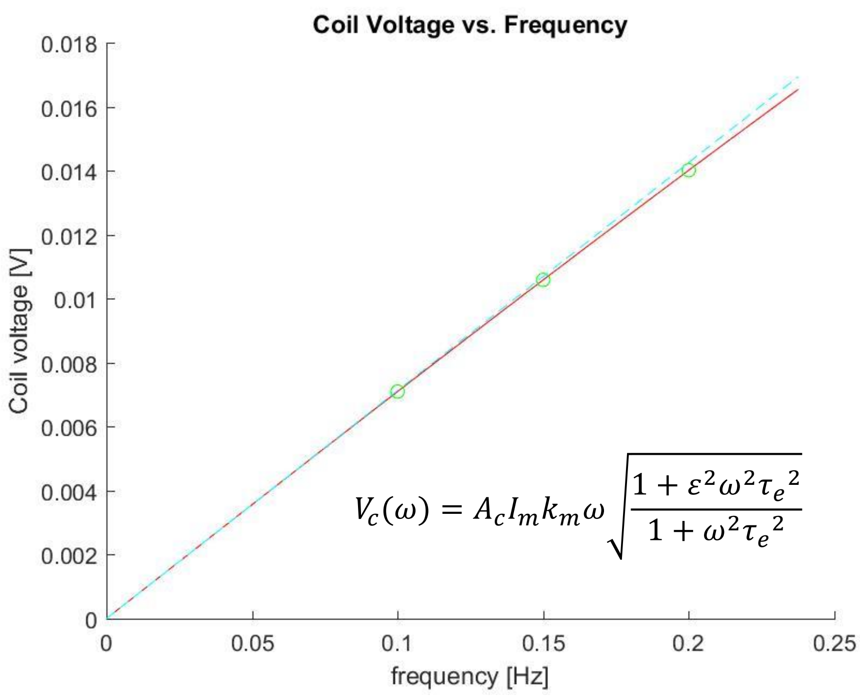
- if the geometry of eddy circuit and magnet coils are similar $\rightarrow \eta = \frac{\lambda_{em}}{\lambda_e} \approx 1$
- if we are far from the magnet $\rightarrow \gamma \approx 1 \rightarrow \varepsilon \approx 0$

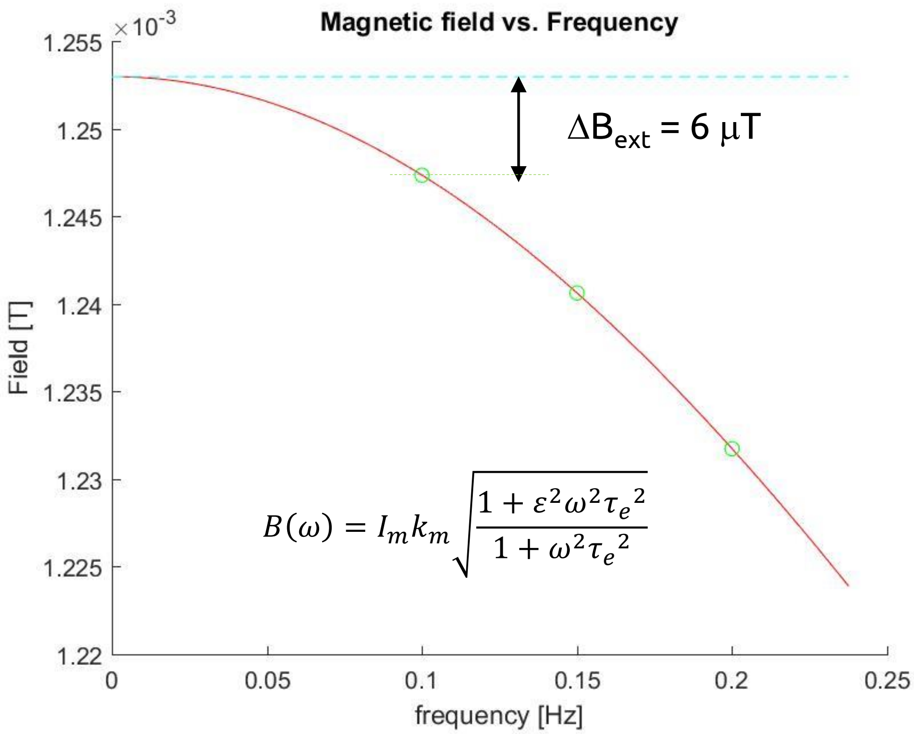
$$\tau_e \frac{dI_e}{dt} + I_e = -\tau_{em}\frac{dI_m}{dt}$$

$$\frac{I_e}{I_m}(s) = -\eta N_t \frac{s\,\tau_e}{1+s\tau_e} \qquad \frac{V_c}{I_m}(s) = A_c k_m s \frac{1+\varepsilon s\tau_e}{1+s\tau_e} \qquad \frac{B}{I_m}(s) = k_m \frac{1+\varepsilon s\tau_e}{1+s\tau_e}$$

Fitting the model to the data

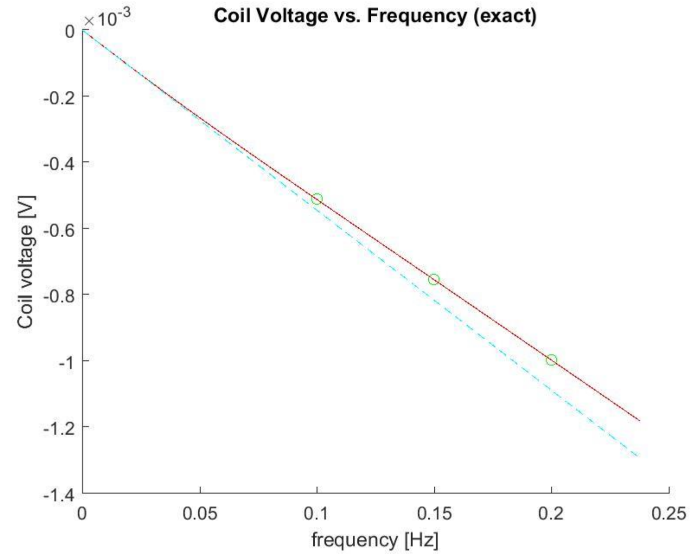
- WP11 measurements: 3 × coil voltages @ 0.10, 0.15 and 0.20 Hz
- Considering amplitudes <u>only</u>, 3 unknowns: k_m, ε and τ_e → model can be solved <u>exactly</u>
  Extrapolated magnetic field value at DC: $B(0) = I_m k_m$
- Sign of the voltage (derived from phase difference w.r.t. excitation current) is taken into account
- The uncertainty of the extrapolated value is taken simply as the RMS of the three measured values
- All this applies to the (large majority of) well-behaved cases …

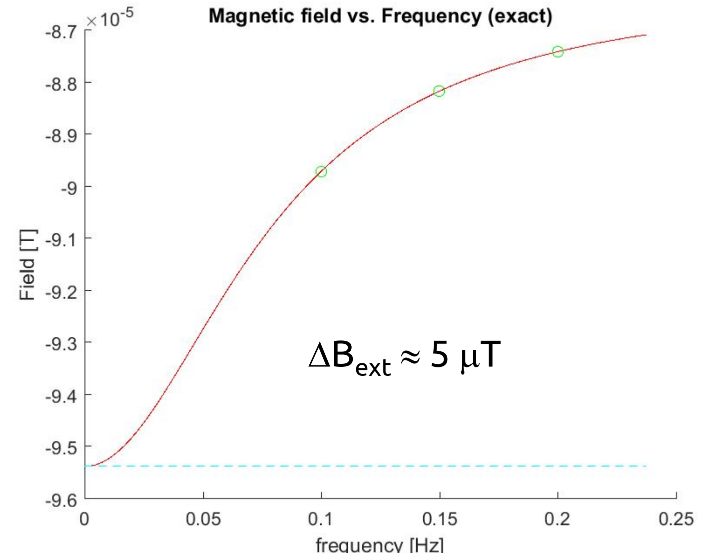




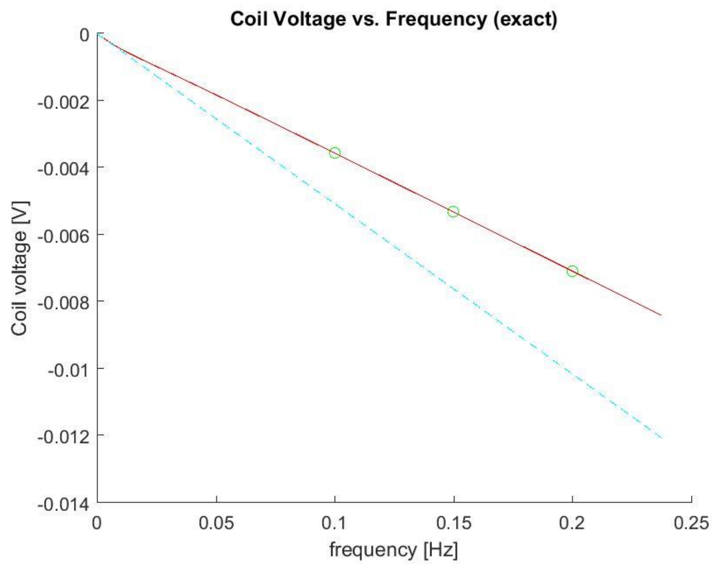
Pathological cases

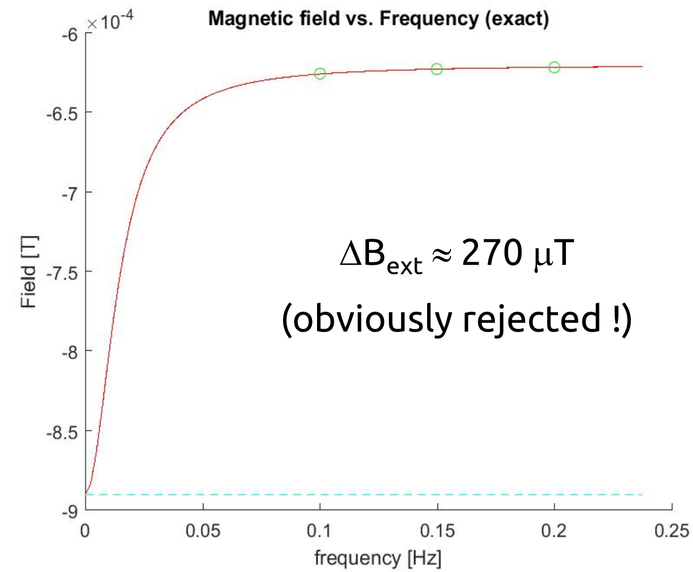
believable ? maybe …





… or maybe not !

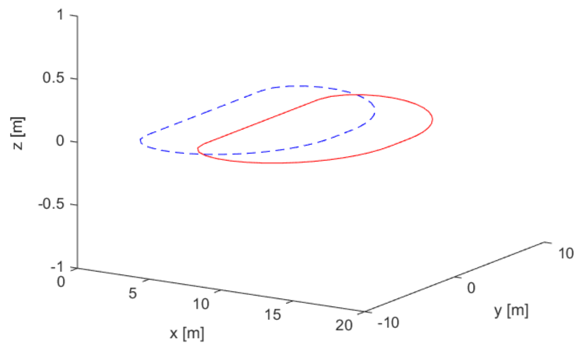




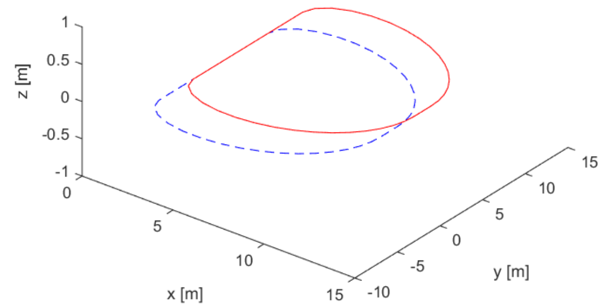
Cause: stiff (and quite complicated) set of algebraic equations, small measurement errors can sometimes be magnified

CCL reconstruction Method

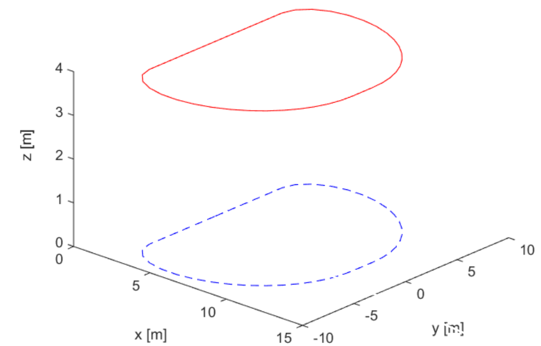
Rigid-body modes



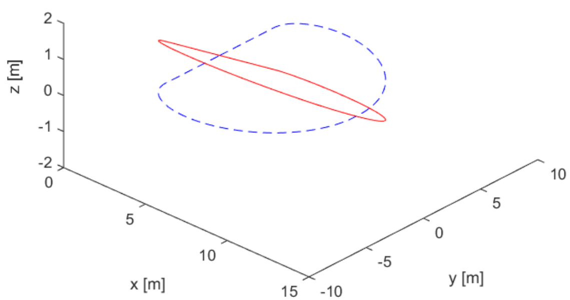
Translation dX



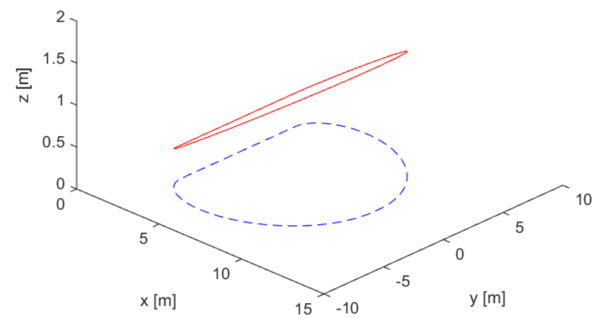
Translation dY



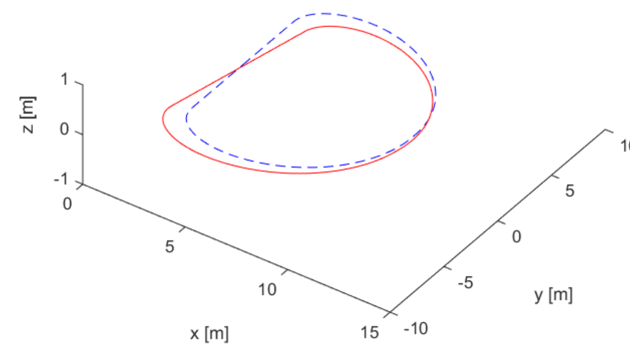
Translation dZ



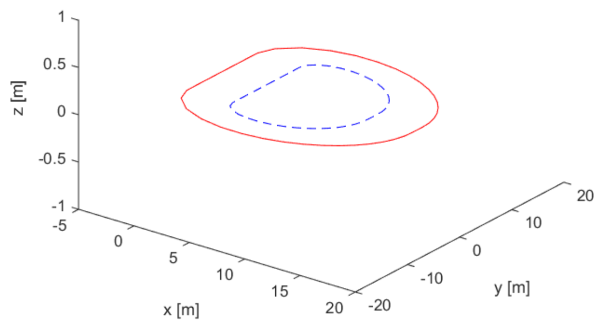
Rotation around X



Rotation around Y

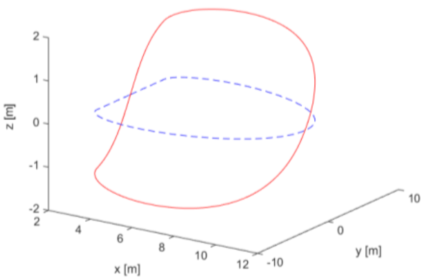


Rotation around Z

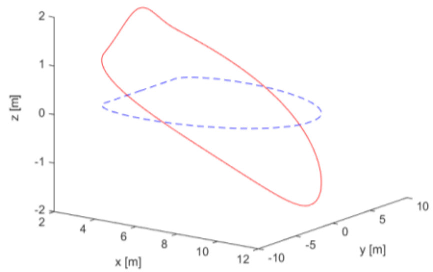


Homothety dR

Harmonic modes



$dZ \propto \sin(2\pi\ \theta)$



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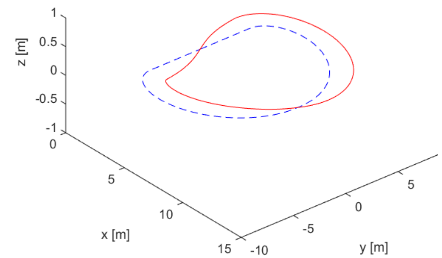

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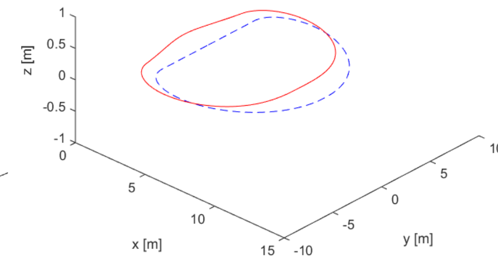

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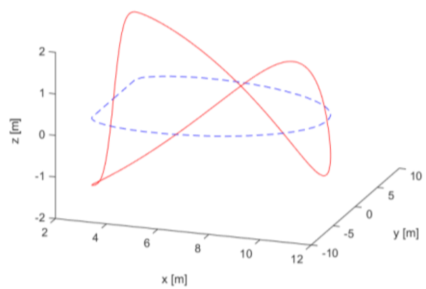
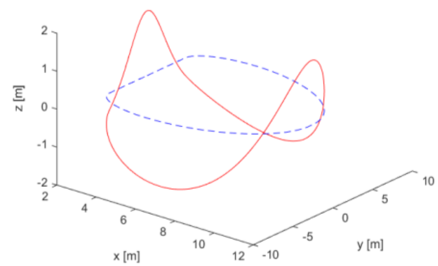


$dZ \propto \sin(2 \cdot 2\pi\ \theta)$



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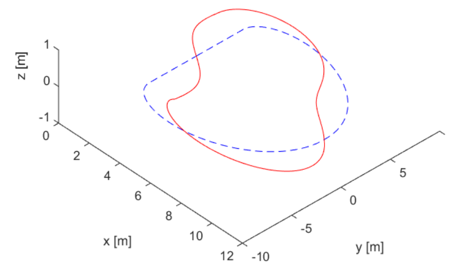

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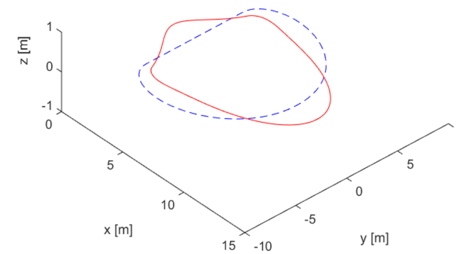
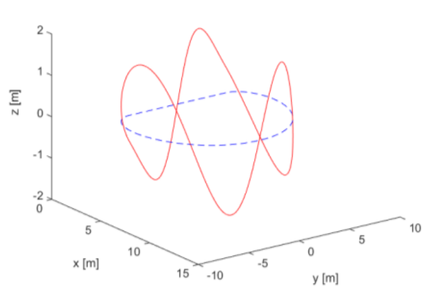
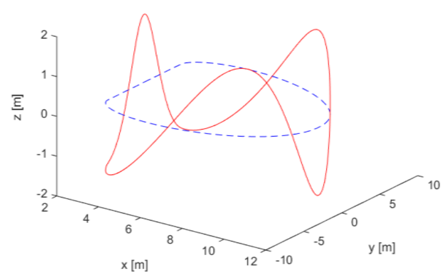


$dR \propto \cos(2 \cdot 2\pi\theta)$



$dZ \propto \sin(3 \cdot 2\pi\theta)$



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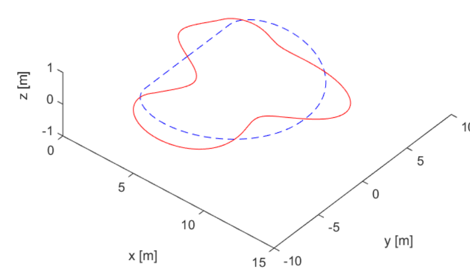
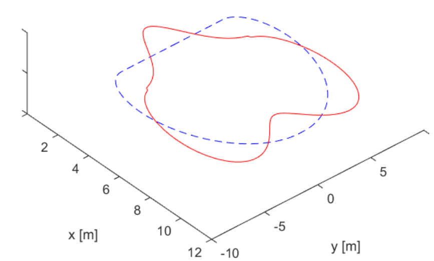


$dR \propto \sin(3 \cdot 2\pi\theta)$



$dR \propto \cos(3 \cdot 2\pi\theta)$

Weighted Least-Squares Parameter Estimation

objective function to be minimized with two (not much) different methods

$$\chi^2(\boldsymbol{\delta}) = \sum_k \frac{\left(B_k^{meas} - B_k(\boldsymbol{\delta})\right)^2}{\sigma_{B_k}^2} = (\boldsymbol{W}\Delta\boldsymbol{B})'(\boldsymbol{W}\Delta\boldsymbol{B}) \approx \chi^2(\boldsymbol{\delta}_0) + \nabla\chi^2\Delta\boldsymbol{\delta} + \frac{1}{2}\Delta\boldsymbol{\delta}'\boldsymbol{H}\Delta\boldsymbol{\delta} + \ldots$$

$$\boldsymbol{J} = \left[\frac{\partial B_k}{\partial \delta_j}\right]$$

$M \times N$
magnetic field Jacobian matrix

$$\boldsymbol{W} = diag\left[\frac{1}{\sigma_{B_k}}\right]$$

$M \times M$
weight matrix
(inverse uncertainty

$$\Delta\boldsymbol{B} = \boldsymbol{B}^{meas} - \boldsymbol{B}(\boldsymbol{\delta})$$

$M \times 1$
measured-computed field difference vector

$$\nabla\chi^2 = -\sum_k \frac{B_k{}^{meas} - B_k(\boldsymbol{\delta})}{\sigma_{B_k}^2}\frac{\partial B_k}{\partial \delta_j} = \boldsymbol{W}\Delta\boldsymbol{B}\,\boldsymbol{W}\boldsymbol{J}$$

$M \times 1$
gradient vector

$$\boldsymbol{H} = \left[\frac{\partial^2\chi^2}{\partial\delta_i\partial\delta_j}\right] \approx (\boldsymbol{W}\boldsymbol{J})'\boldsymbol{W}\boldsymbol{J}$$

$N \times N$
Hessian matrix
(omitting by default second derivative terms)

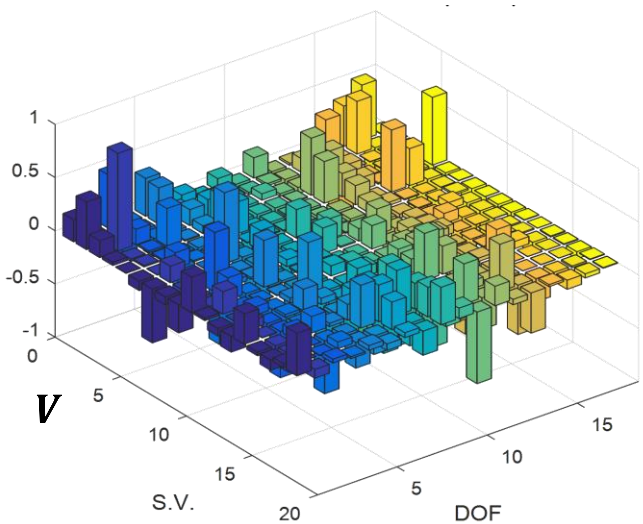
find iteratively the d.o.f. increment $\Delta\delta$ that satisfies:

(linear approximation) $\chi^2(\boldsymbol{\delta}) = 0 \Rightarrow 0 = -\boldsymbol{W}\Delta\boldsymbol{B}(\boldsymbol{\delta}_0) + \boldsymbol{W}\boldsymbol{J}\,\Delta\boldsymbol{\delta} \Rightarrow \Delta\boldsymbol{\delta} = (\boldsymbol{W}\boldsymbol{J})^\dagger\Delta\boldsymbol{B}(\boldsymbol{\delta}_0)$

(quadratic approximation) $\min\chi^2(\boldsymbol{\delta}) \Rightarrow 0 = -\boldsymbol{W}\boldsymbol{J}\Delta\boldsymbol{B}(\boldsymbol{\delta}_0) + \boldsymbol{H}\,\Delta\boldsymbol{\delta} \Rightarrow \Delta\boldsymbol{\delta} = \boldsymbol{H}^\dagger\boldsymbol{W}\boldsymbol{J}\Delta\boldsymbol{B}(\boldsymbol{\delta}_0)$

$$(\boldsymbol{WJ}\ or\ \boldsymbol{H}) = \boldsymbol{U}\,diag[s_j]\boldsymbol{V}$$

$$(\boldsymbol{WJ}\ or\ \boldsymbol{H})^\dagger = \boldsymbol{V}'diag\left[\begin{cases} if\ \frac{s_j}{s_1} > tol, & \frac{1}{s_j} \\ if\ \frac{s_j}{s_1} \le tol, & 0 \end{cases}\right]\boldsymbol{U}'$$



identify linear combinations of d.o.f. that correspond to small singular values and degrade system conditioning

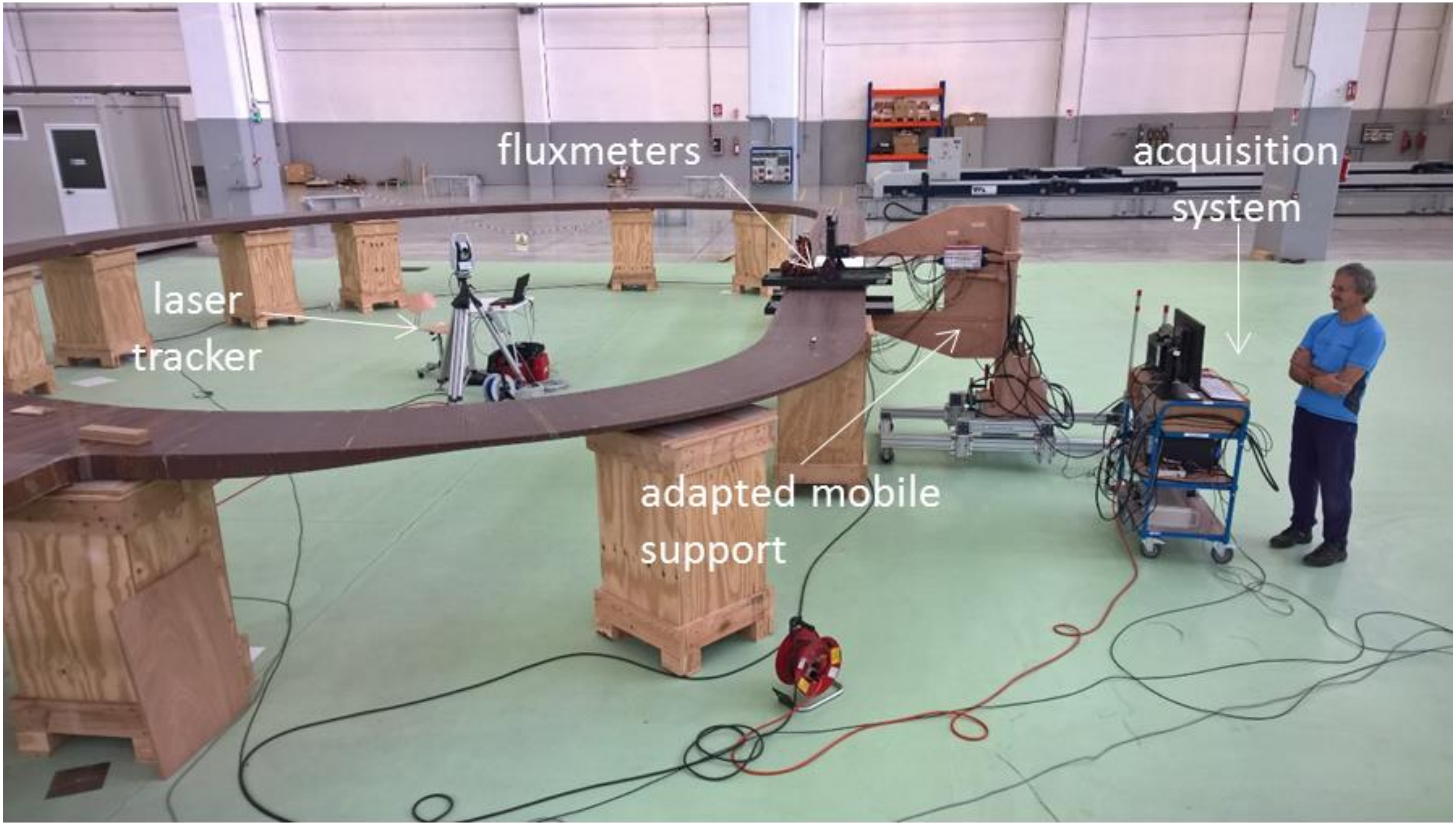
variance of estimated parameters:

$$\sigma_j^2 = diag(\boldsymbol{H}^{-1})$$

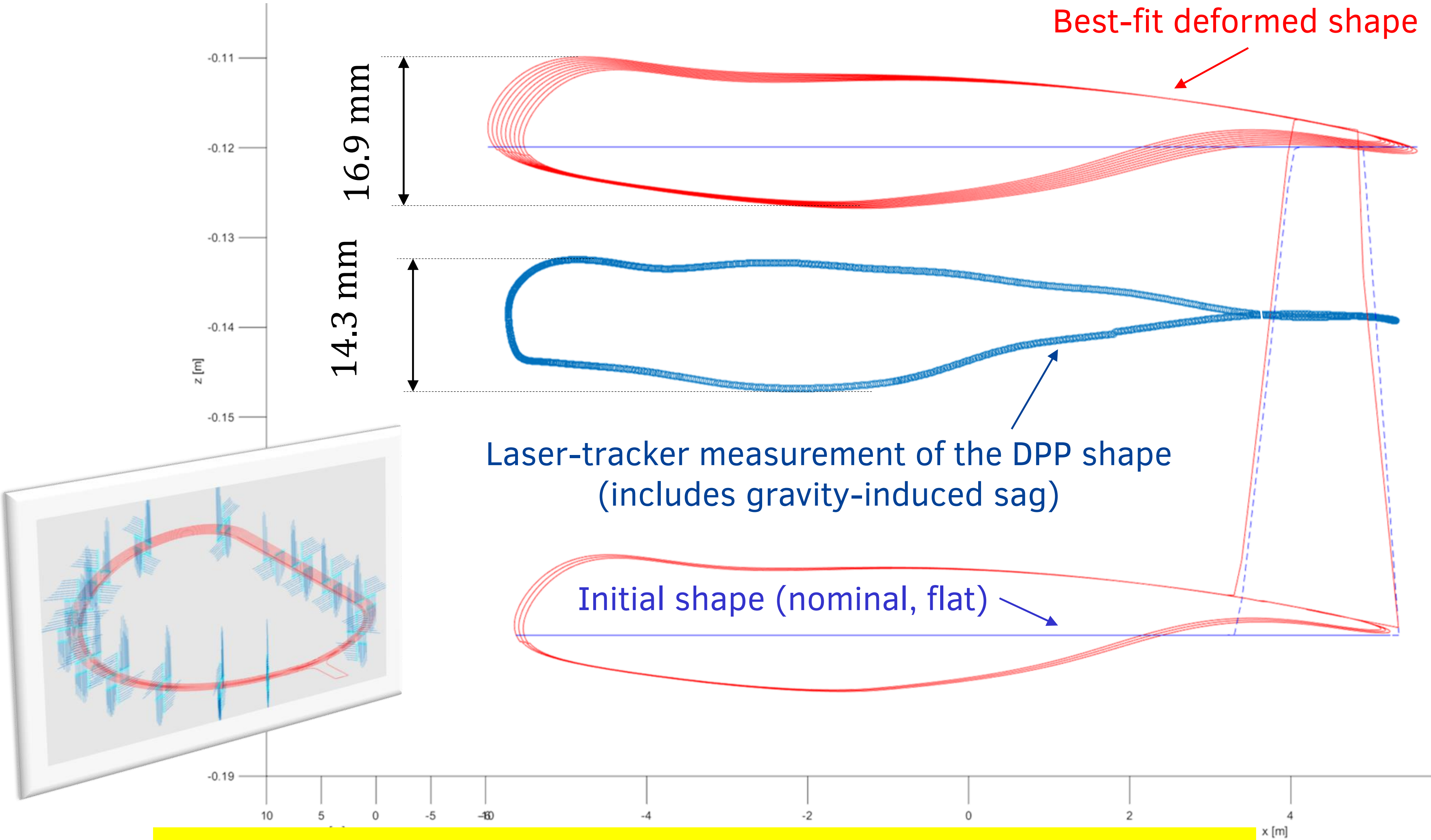
correlation terms ignored up to now $\Rightarrow$ possibile overstimation of error bars

Dual Pancake Prototype test & results

Dual Pancake Prototype



DPP test result

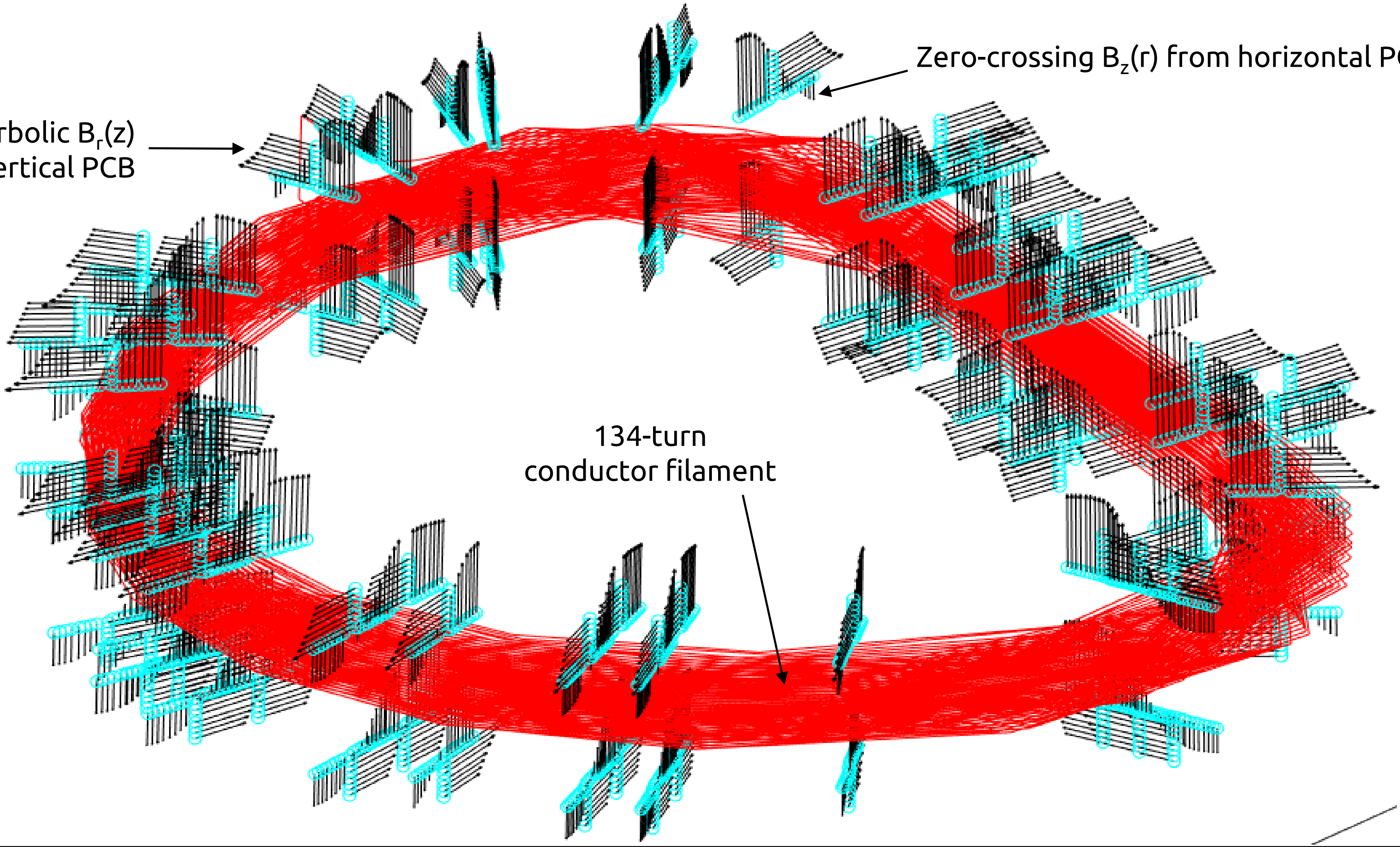


3D deformation measured magnetically matches geometrical survey

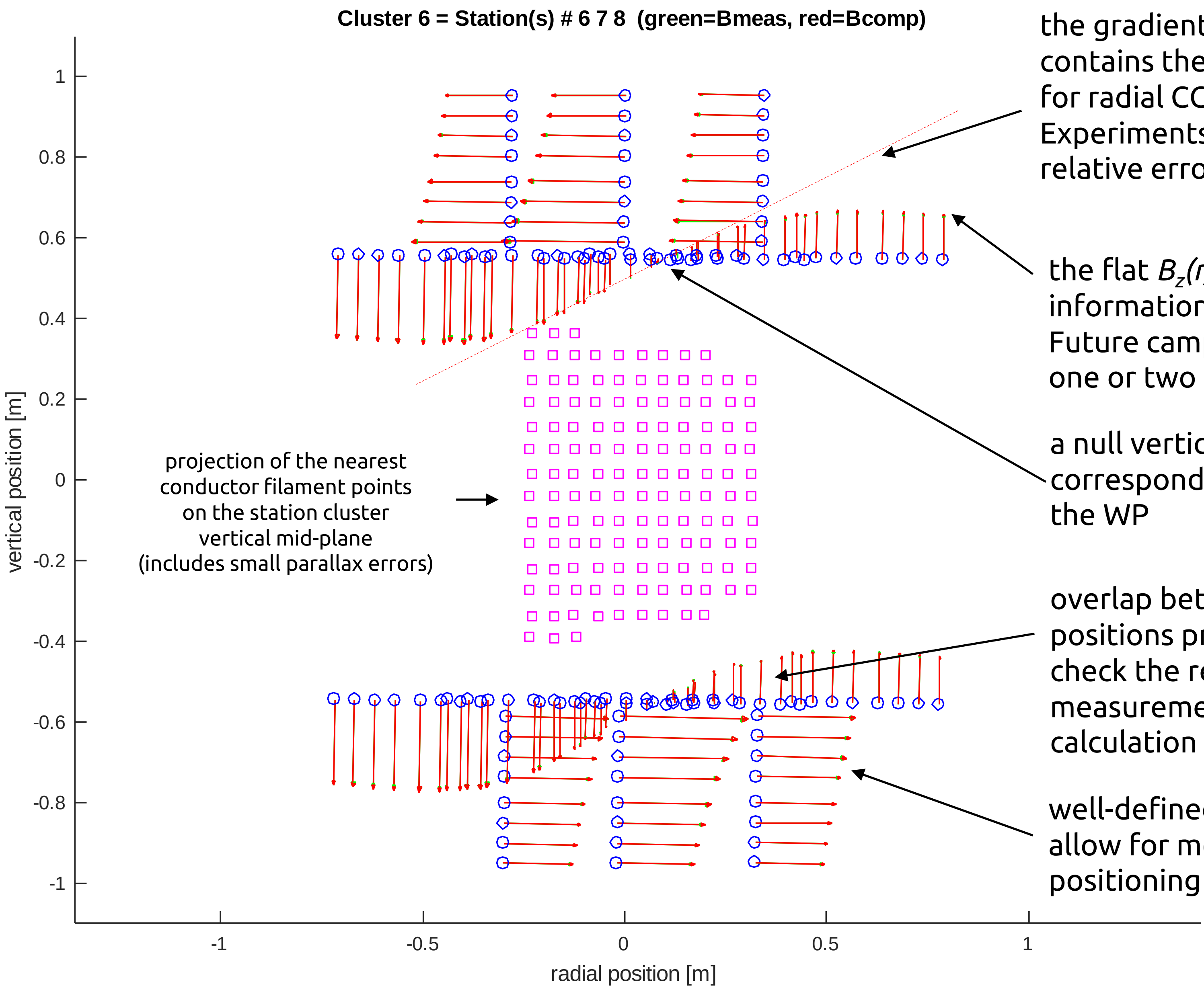
Winding Pack 11 test & results

Field map

- **7200** data points @ 35~38 A = 48 pick-up coils × 50 stations × (0.1, 0.15 and 0.2 Hz)
- Max. measured field: **1640 μT** (radial), 1360 μT (vertical)
- Max. measured gradient: 2.7 μT/mm (radial), **3.4 μT/mm** (vertical)



Local field map example



the gradient at the zero-crossing contains the information necessary for radial CCL positioning. Experiments show there large relative errors.

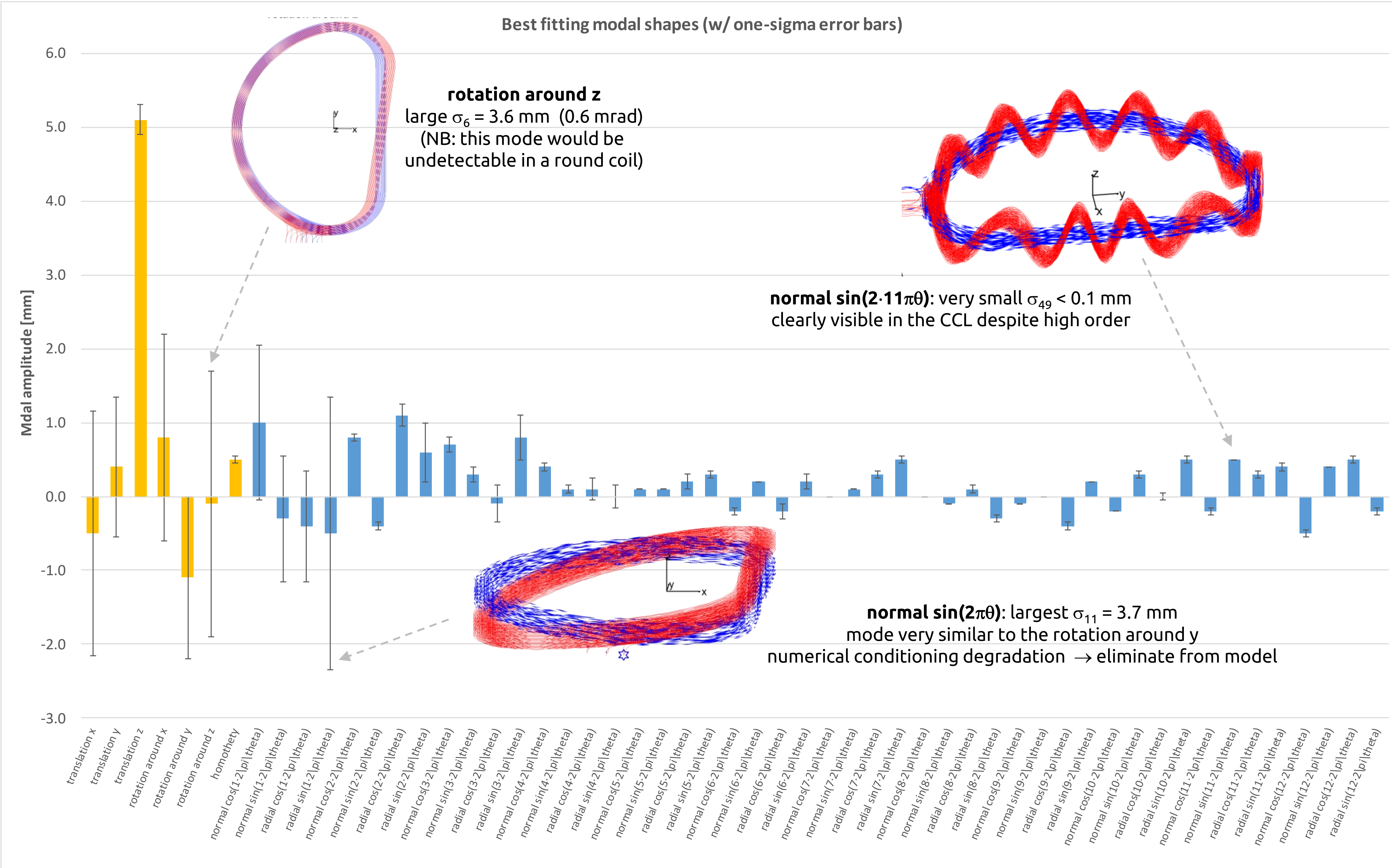
the flat $B_z(r)$ profile contains no useful information for the radial positioning. Future campaigns should be limited to one or two radially shifted stations

a null vertical field component corresponds roughly to the center of the WP

overlap between radially shifted positions provides the opportunity to check the repeatability of the measurement (interpolation needed, calculation pending)

well-defined hyperbolic *Br(z)* profiles allow for more precise vertical CCL positioning

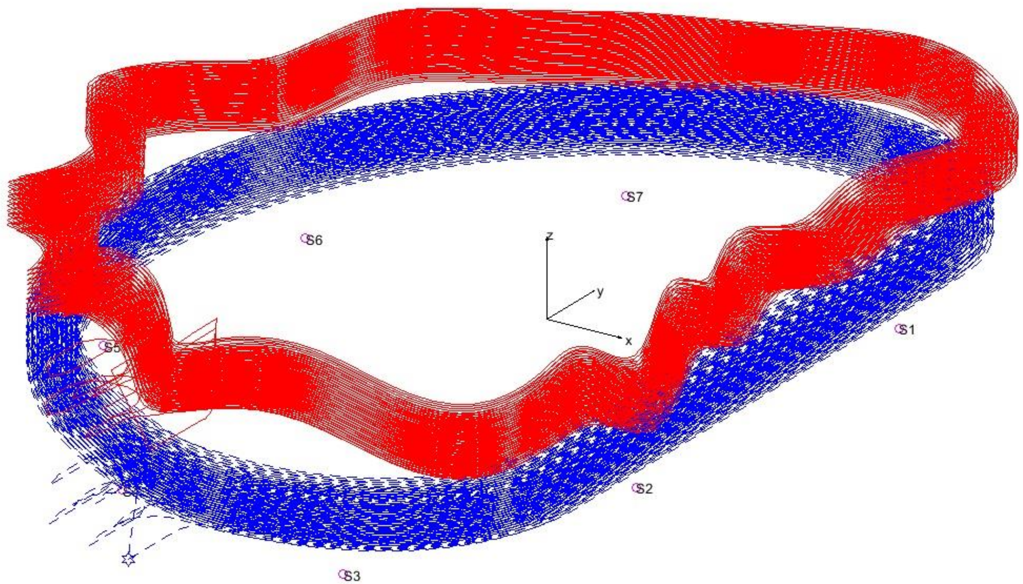
Best-fit results



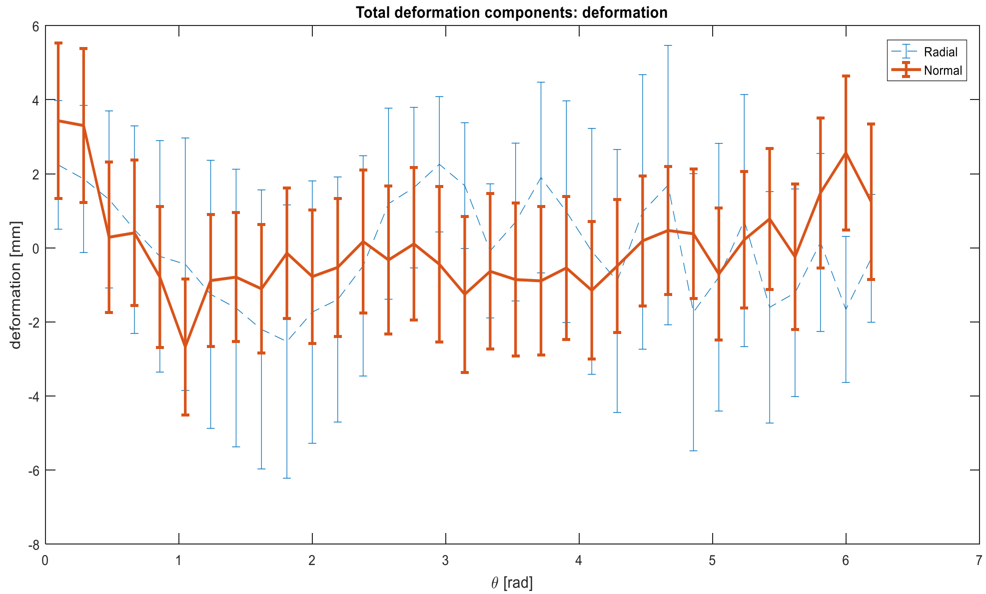
Reconstructed CCL

- First stable result of WP11 CCL reconstruction
- Conductor discretized with 21300 points (220 mm step, 160 segments/turn)
- Biot-Savart computation on a 5 × 3 grid on each measuring coil
- Shape defined by **55 d.o.f.**: 6 rigid-body + homothety + harmonics up to ***n*=12** (λ = 2.8 m)
- Excitation current included as a variable to compensate for systematic coil area error

- Max CCL deformation (excl. rigid-body modes) **4.1 mm**
- RMS deformation uncertainty: **±2 mm normal**, **±3 mm radial**



deformation exaggerated for visualization
(harmonic components only)



radial (horizontal) and normal (vertical) deformation at discrete CCL points
azimuth θ counterclockwise starting from connection region
no clear difference between straight and curved regions

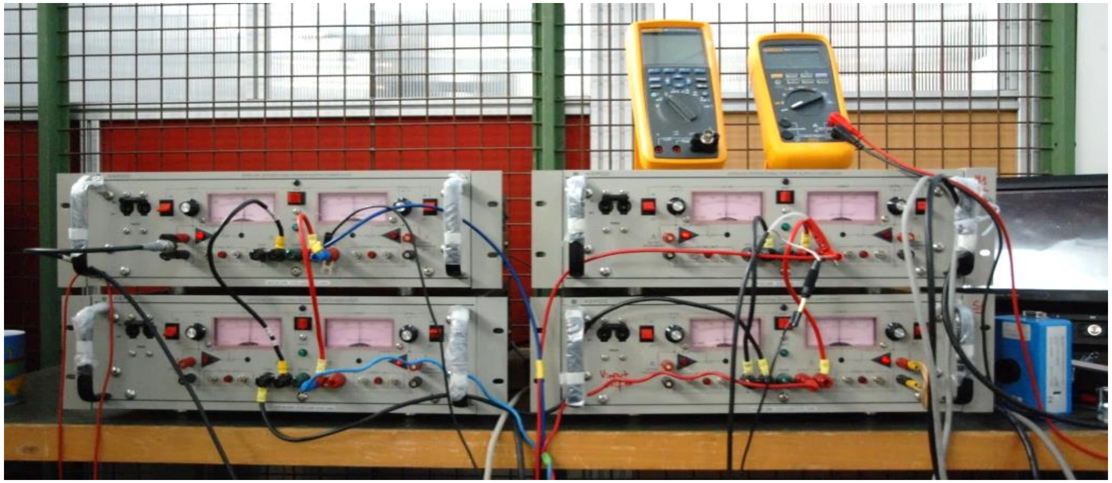
Conclusions

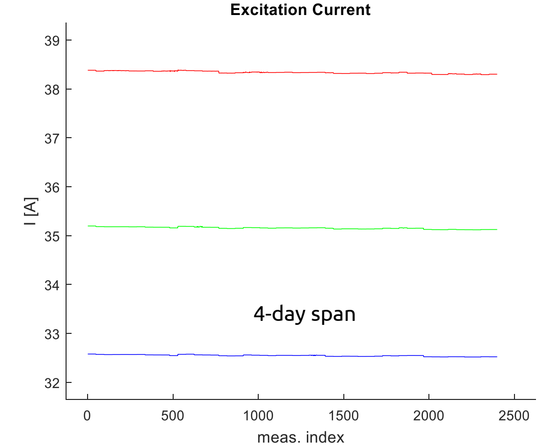
Conclusions

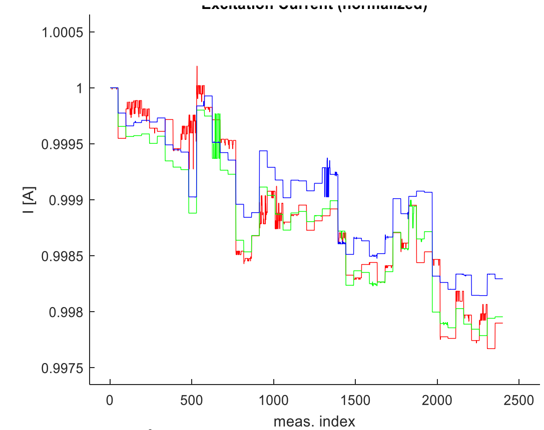
- The delivered instrument provides a **magnetic measurement uncertainty ≈ $3{\cdot}10^{-4}$** very good metrological performance at a level of few mT
- Early concerns such as eddy current effects proven to be manageable
- System + reconstruction method are capable to reconstruct the position of the CCL within an **uncertainty of 4 mm** (radial direction) and **6 mm** (vertical direction) (at one sigma)
- Uncertainty analysis → clear indications towards feasible improvements
- Further tests planned to evaluate continuation of series WP or encased TF measurements and possibly switching to the mapping technique

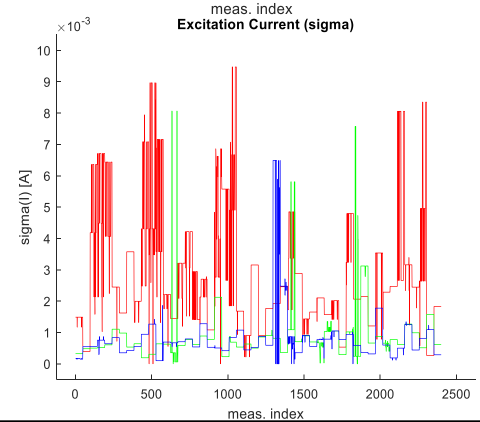
Additional slides

AC excitation current





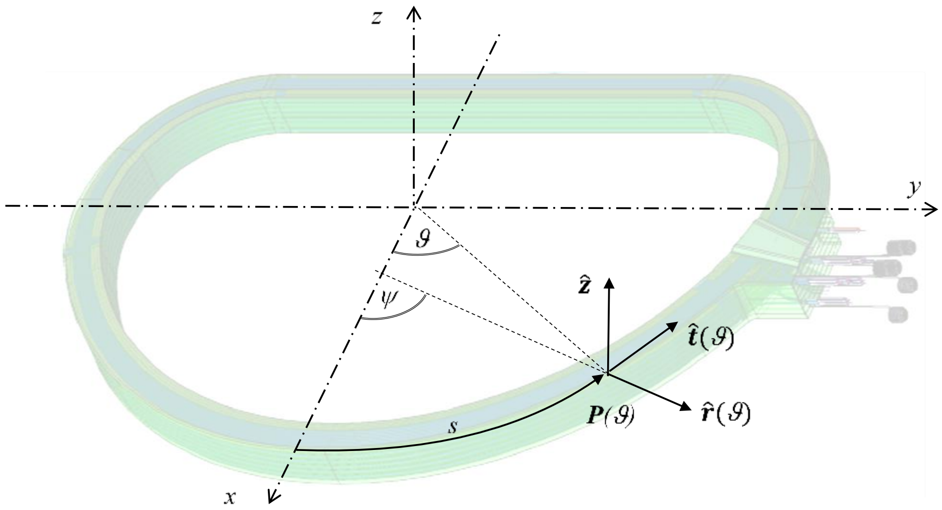


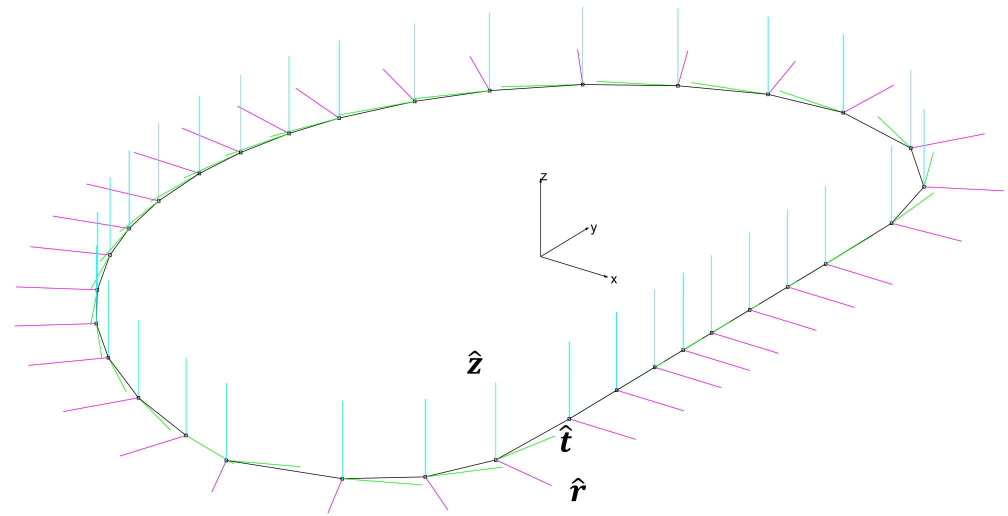


f_{AC} [Hz]	Current		Coil Output		
	Average [A]	Repeatability [A RMS]	Min. [μV]	Max. [mV]	Repeatability [μV RMS]
0.10	38.337	0.026	20	10.3	0.3
0.15	35.155	0.021	39	14.1	0.4
0.20	32.547	0.017	60	17.3	0.3

- Series/parallel combination of 4 KEPCO 20/20 power supplies
- ~ one day to find stable working points on WP load at ASG
- Short-term stability 10 mA ($3 \cdot 10^{-4}$), long-term 100 mA ($3 \cdot 10^{-3}$)
- Measured with PM zero-flux DCCT MACC Plus 100A/10V ($<10^{-5}$)
- Inexpensive but fiddly setup, difficult to fine-tune even with help from Kepco experts
- Not scalable and reliable enough for series tests

Modal decomposition of the CCL shape





$$\boldsymbol{P}_i(\boldsymbol{\delta}) = \boldsymbol{P}_{0,i} + \sum_{j=1}^{n} \delta_j \boldsymbol{\varphi}_j(\vartheta_i), \qquad i = 1..N_P$$

$$\boldsymbol{P}_i(\boldsymbol{\delta}) = \boldsymbol{P}_{0,i} + \sum_{j=1}^{n} \delta_j \boldsymbol{\varphi}_j(\vartheta_i), \qquad i = 1..N$$

$$\boldsymbol{\varphi}_j(\vartheta) = \varphi_j^r(\vartheta)\hat{\boldsymbol{r}}(\vartheta) + \varphi_j^z(\vartheta)\hat{\boldsymbol{z}} + \varphi_j^t(\vartheta)\hat{\boldsymbol{t}}$$

$$\max\|\boldsymbol{\varphi}_j\| = 1$$

normalized modal shapes (all expressed in mm)

harmonic modes

$$\begin{cases} \varphi_j^r(\vartheta, n) = a_j^r \sin 2\pi n\vartheta + b_j^r \cos 2\pi n\vartheta \\ \varphi_j^z(\vartheta, n) = a_j^z \sin 2\pi n\vartheta + b_j^z \cos 2\pi n\vartheta \end{cases}$$

$$a_j^{r,z} = \begin{cases} j\ even & 1 \\ j\ odd & 0 \end{cases} \qquad b_j^{r,z} = \begin{cases} j\ even & 0 \\ j\ odd & 1 \end{cases}$$

translations

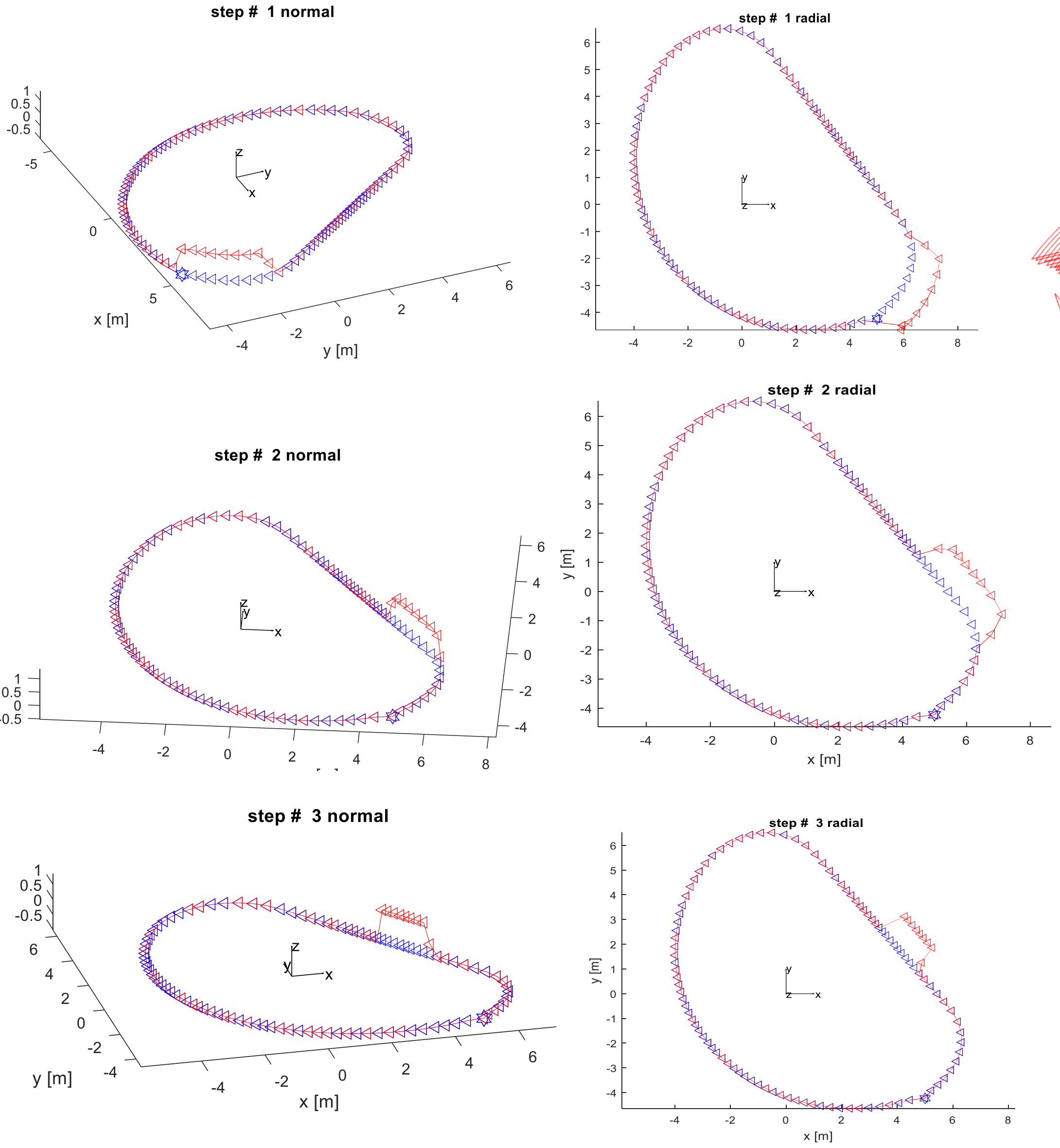
$$\begin{cases} \boldsymbol{\varphi}_x(\vartheta) = \cos\psi(\vartheta)\hat{\boldsymbol{r}}(\vartheta) + \sin\psi(\vartheta)\hat{\boldsymbol{t}} \\ \boldsymbol{\varphi}_y(\vartheta) = -\sin\psi(\vartheta)\hat{\boldsymbol{r}}(\vartheta) + \cos\psi(\vartheta)\hat{\boldsymbol{t}} \\ \boldsymbol{\varphi}_z(\vartheta) = \hat{\boldsymbol{z}}(\vartheta) \end{cases}$$

homothety

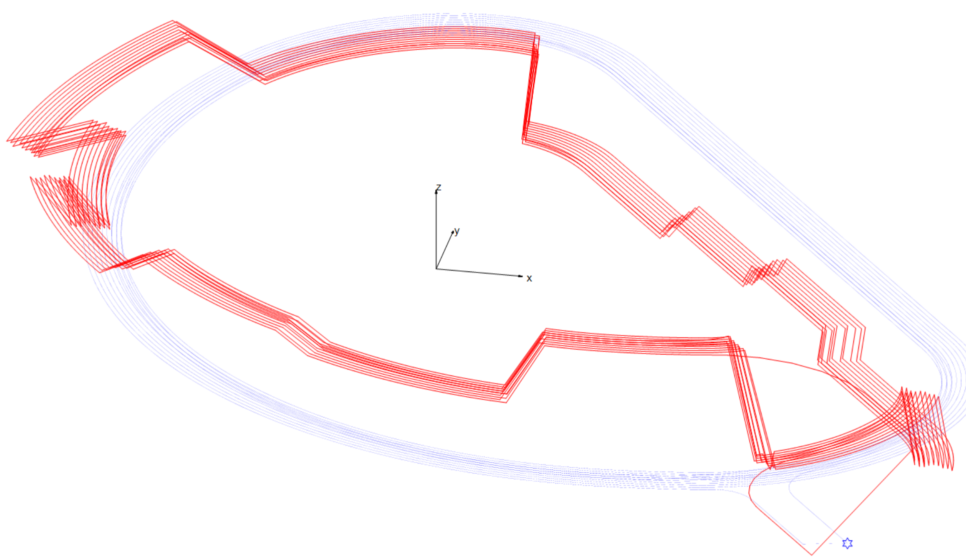
$$\boldsymbol{\varphi}_h(\vartheta) = \hat{\boldsymbol{r}}(\vartheta)$$

rotation

$$\begin{cases} \boldsymbol{\varphi}_{rx}(\vartheta) = y\,\hat{\boldsymbol{z}}(\vartheta) \\ \boldsymbol{\varphi}_{ry}(\vartheta) = x\,\hat{\boldsymbol{z}}(\vartheta) \\ \boldsymbol{\varphi}_{rz}(\vartheta) = x\cos\vartheta\,\hat{\boldsymbol{t}}(\vartheta) - y\cos\vartheta\,\hat{\boldsymbol{r}}(\vartheta) \end{cases}$$



(...)

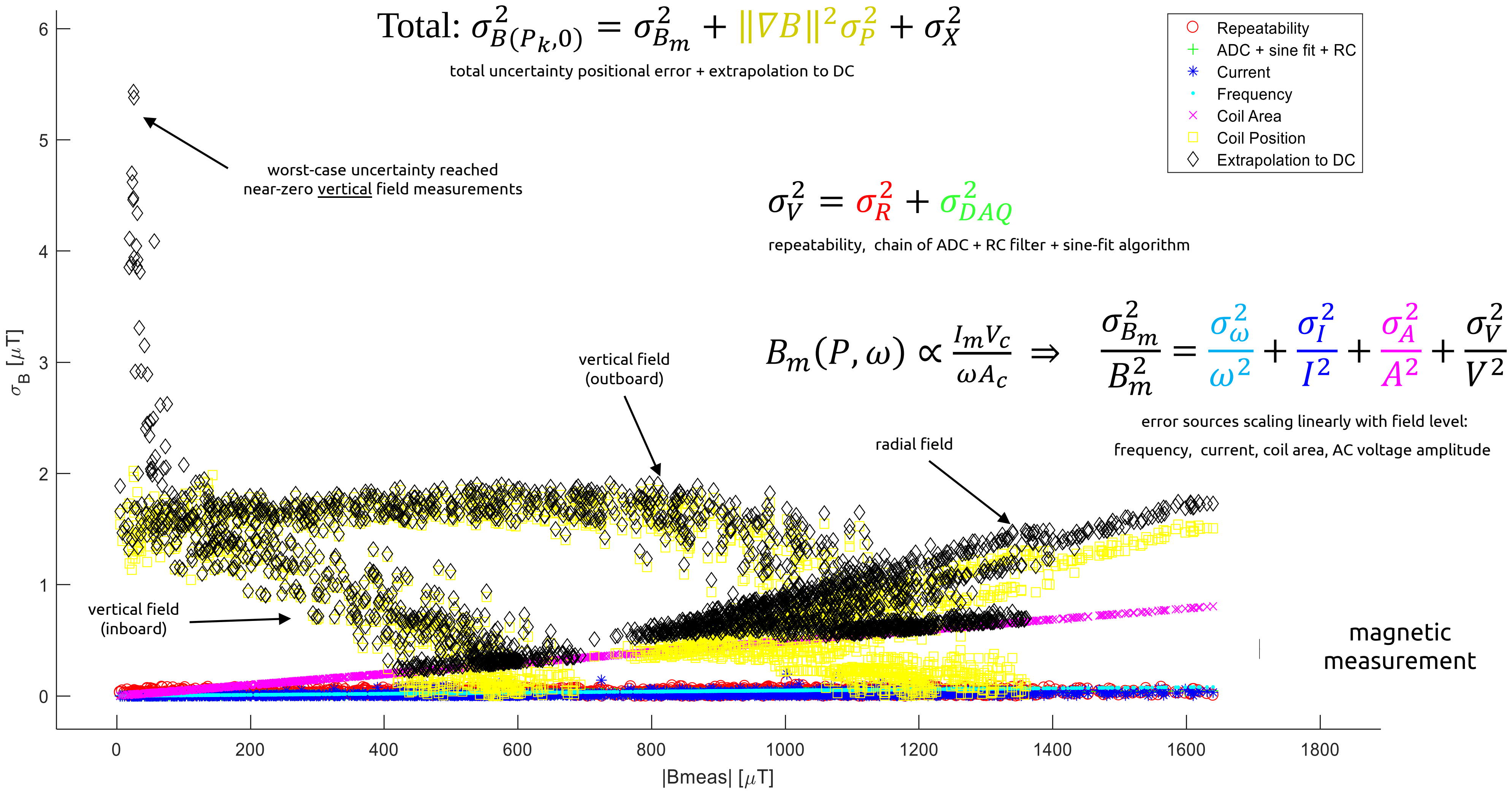


Example of CCL reconstruction result with stepwise constant modes

physically inconsistent, but might be more robust to model strongly localized deformations

Measurement uncertainty

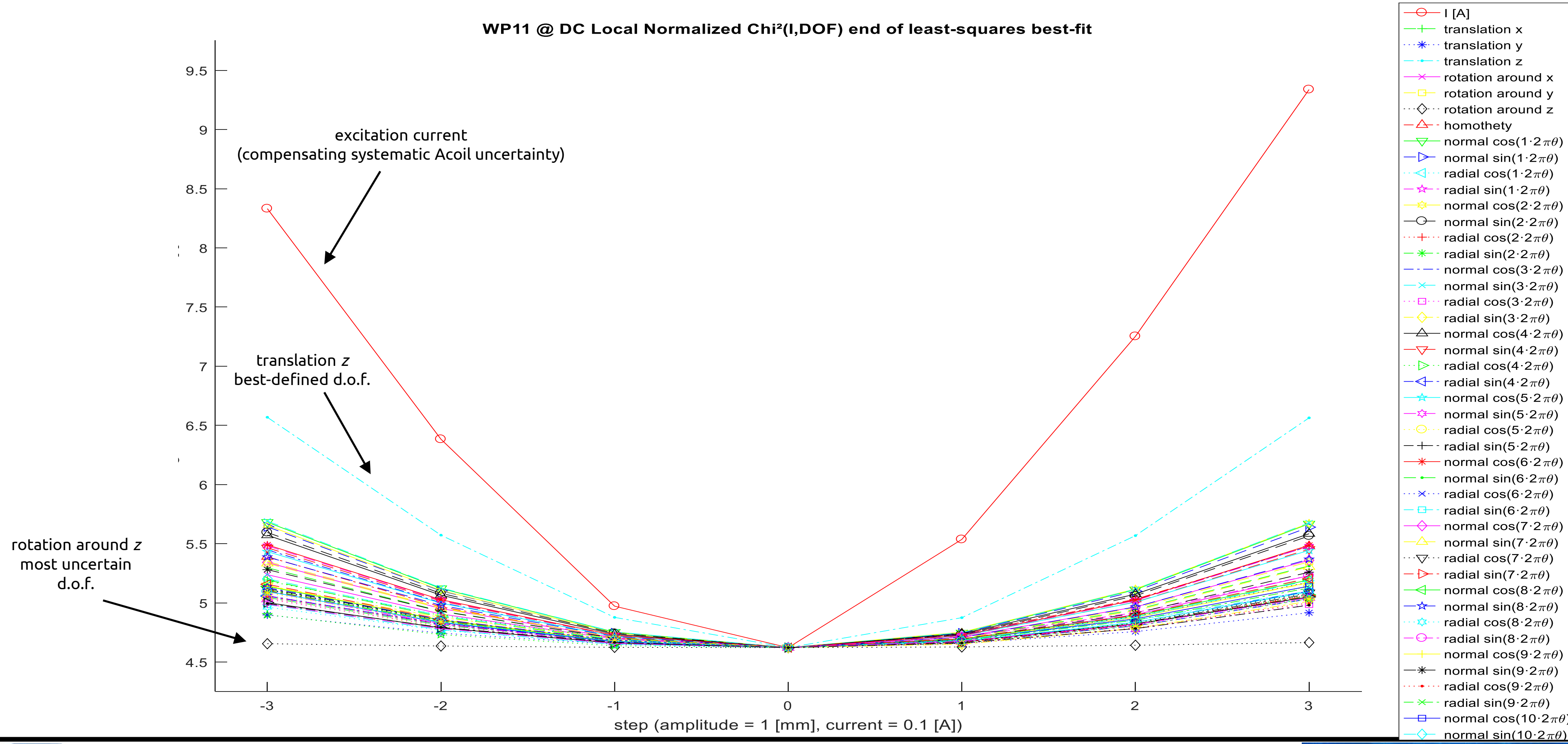
- Uncertainty of the magnetic measurement alone **< 0.5 μT** (~3·10^{-4} of full range, all frequencies)
- Estimated total measurement uncertainty generally ≤ 2 μT (~10^{-3} of full scale)
- Dominant terms: coil position error (0.8 mm RMS), coil area (5.5·10^{-4})



Have we really found a minimum ?

- Hessian matrix positive definite ⇒ guaranteed local minimum
- **Normalized χ^2 = 4.6** reasonably close to expected unit value
- RMS field difference = **5 μT** is best result so far in *numerous* trial-and error attempts

⇒ we are probably close to the global minimum (for this particular dataset)



Best-fit quality

Agreement between calculation and measurements improved 3× by the minimization:

	(initial best guess)		(end of least-squares iterations)	
• RMS field difference	**14 μT**	→	**5 μT**	
	0.8 %	→	**0.3 %**	(w.r.t. full scale 1.6 mT)
• Normalized χ^2	**13.6**	→	**4.6**	
• Correlation slope	**1.009**	→	**0.99998**